The Dispersion of Slepian-Wolf Coding

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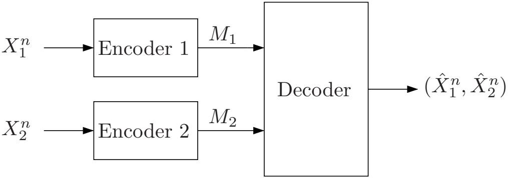
Joint work with

Oliver Kosut
Massachusetts Institute of Technology

The Slepian-Wolf Problem

The optimal rate region for the Slepian-Wolf problem $\mathcal{R}^*_{\mathrm{SW}}$ is



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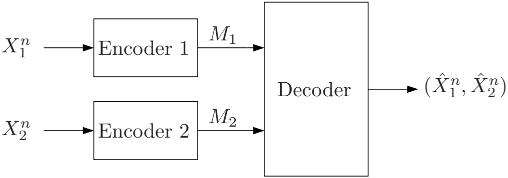

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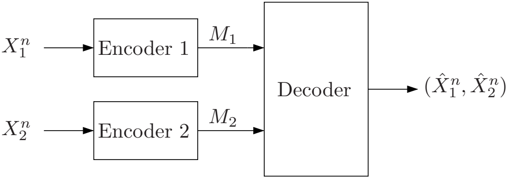

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The (n, ϵ)-region $\mathcal{R}^*_{\mathrm{SW}}(n, \epsilon)$ is the set of all (n, ϵ)-achievable rates.

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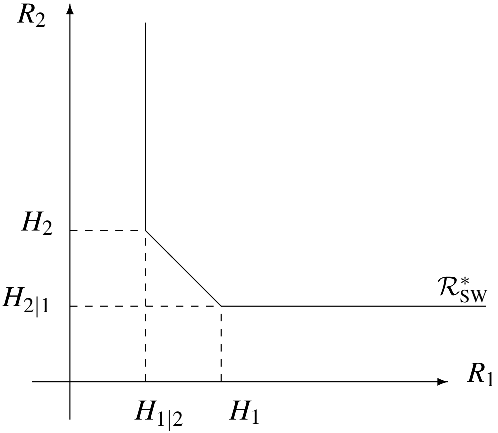

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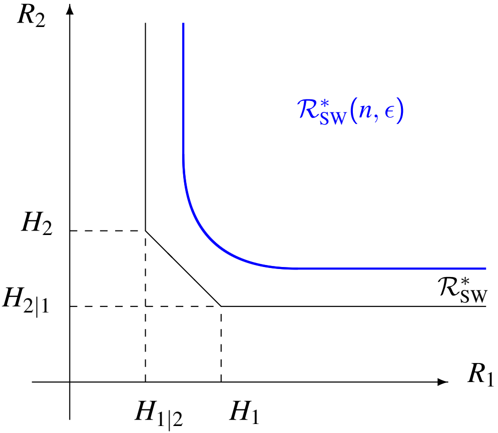

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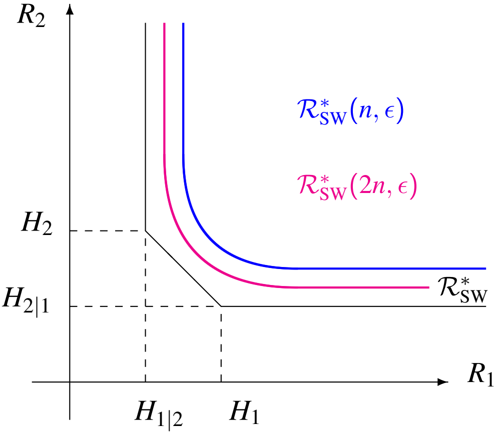

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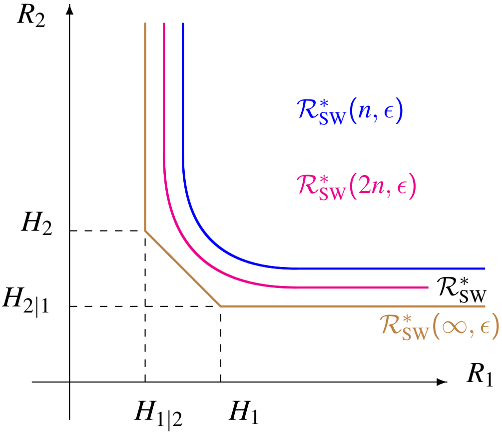

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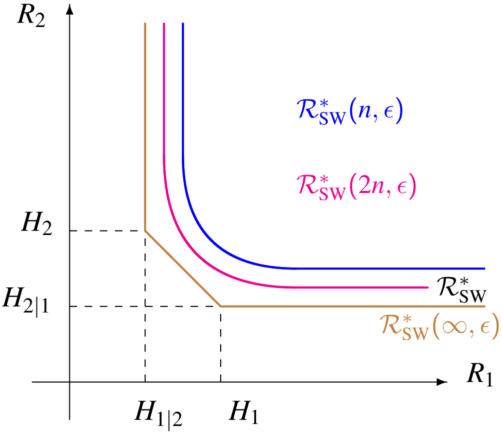

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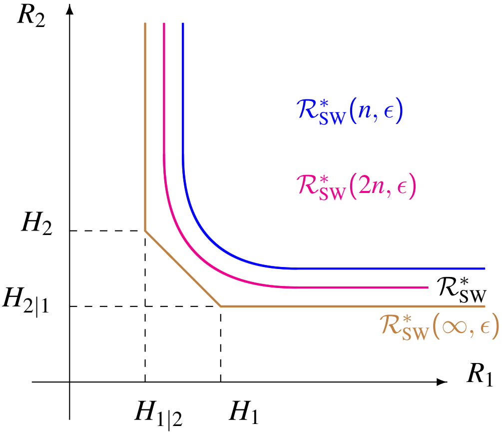

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Figure "correct" up to $O(\frac{\log n}{n})$ terms.

Prior Work on (n, ϵ)-regions

- For fixed-length lossless source coding [Strassen (1964), Kontoyiannis (1997), Hayashi (2008)],

$$R^*(n,\epsilon) = H(X) + \sqrt{\frac{V_1}{n}} Q^{-1}(\epsilon) + O\left(\frac{\log n}{n}\right)$$

where the source coding dispersion $V_1 := \mathrm{Var}(-\log p(X))$.

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- Recent second-order coding rate results for
  1. Intrinsic randomness and resolvability (Hayashi 2008)
  2. Channel coding (Polyanskiy et al. 2010 and Hayashi 2009)
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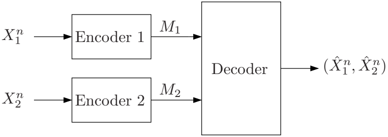
- First non-trivial network information theory problem (where there is more than one rate) is the Slepian-Wolf problem

(n, ϵ)-region for the Slepian-Wolf Problem



An $(n, 2^{nR_1}, 2^{nR_2}, \epsilon)$ code for the source (X_1, X_2) is characterized by

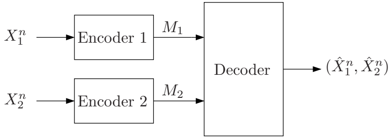
1. Two Encoders: $f_{j,n} : \mathcal{X}_j^n \to [1 : 2^{nR_j}]$ for $j = 1, 2$
2. Decoder: $\varphi_n : [1 : 2^{nR_1}] \times [1 : 2^{nR_2}] \to \mathcal{X}_1^n \times \mathcal{X}_2^n$

such that $\mathbb{P}((\hat{X}_1^n, \hat{X}_2^n) \neq (X_1^n, X_2^n)) \leq \epsilon$.

Definition

Rate pair (R_1, R_2) is (n, ϵ)-achievable if there exists an $(n, 2^{nR_1}, 2^{nR_2}, \epsilon)$ code for the source (X_1, X_2).

(n, ϵ)-region for the Slepian-Wolf Problem



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The (n, ϵ)-optimal rate region $\mathcal{R}^*_{\mathrm{SW}}(n, \epsilon)$ is the set of all (n, ϵ)-achievable rate pairs.

Related Work

- Baron et al. (2004): Slepian-Wolf with perfect side-information

$$R_1 \approx H(X_1|X_2) + \sqrt{\frac{V}{n}} Q^{-1}(\epsilon)$$

where $V = \mathrm{Var}(-\log p_{X_1|X_2}(X_1|X_2))$

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- Sarvotham-Baron-Baraniuk (2005): Slepian-Wolf for correlated Bernoulli(1/2) sources

$$R_1 \geq H(X_1|X_2) + \sqrt{\frac{V_1}{n}} Q^{-1}(\epsilon)$$

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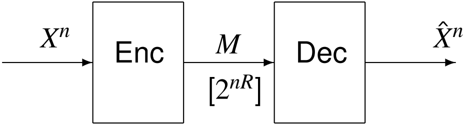
where $V_1 = \mathrm{Var}(-\log p_{X_1|X_2}(X_1|X_2))$, $V_2 = \mathrm{Var}(-\log p_{X_2|X_1}(X_2|X_1))$ and $V_3 = \mathrm{Var}(-\log p_{X_1,X_2}(X_1, X_2))$.

Point-To-Point Source Coding Revisited



Rate R is (n, ϵ)-achievable if [Strassen (1964) and Kontoyiannis (1997)]

$$R \geq H(X) + \sqrt{\frac{V}{n}} Q^{-1}(\epsilon)$$

Define entropy density $h(x) = -\log p_X(x)$, then

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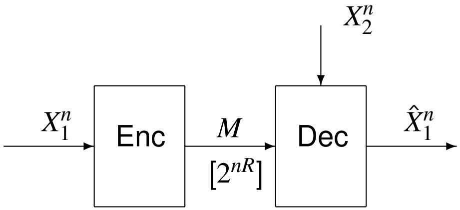
Slepian-Wolf with Perfect Side Information



Baron et al. (2004): Rate R is (n, ϵ)-achievable if

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Define conditional entropy density $h(x_1|x_2) = -\log p_{X_1|X_2}(x_1|x_2)$, then

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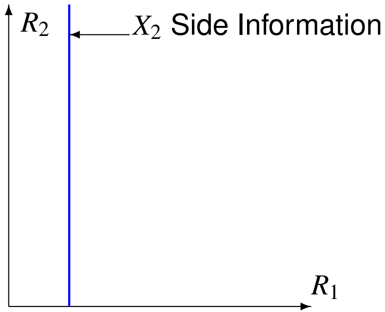
Simple Slepian-Wolf Outer Bound



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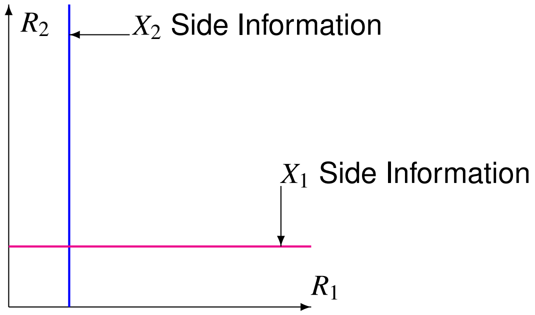

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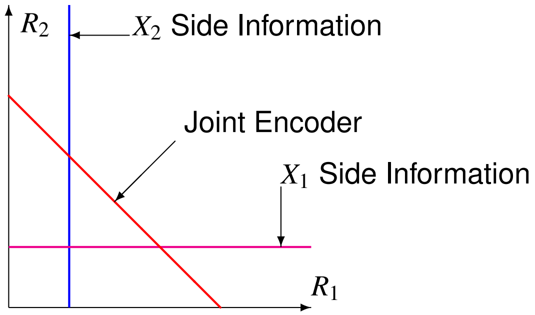

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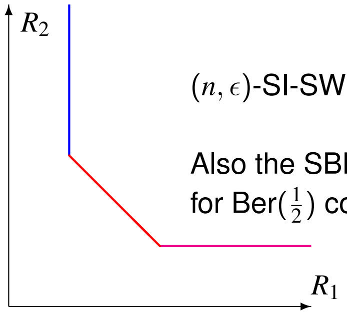

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$$R \geq H(X) + \sqrt{\frac{V}{n}} Q^{-1}(\epsilon) \quad \Leftrightarrow \quad R \in H(X) + \frac{1}{\sqrt{n}} \mathcal{S}_V(\epsilon)$$

where the set

$$\mathcal{S}_V(\epsilon) = \{z \in \mathbb{R} : \mathbb{P}(Z \leq z) \geq 1 - \epsilon\}$$

and the random variable $Z \sim \mathcal{N}(0, V)$.

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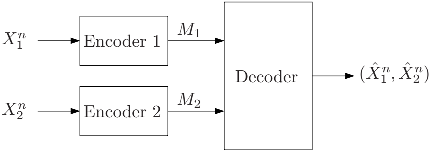
What's the analogue of this result for the two-encoder Slepian-Wolf problem?



Slepian-Wolf Dispersion Result

- Define entropy density vector

$$\mathbf{h}(x_1, x_2) := \begin{bmatrix} -\log p_{X_1|X_2}(x_1|x_2) \\ -\log p_{X_2|X_1}(x_2|x_1) \\ -\log p_{X_1,X_2}(x_1, x_2) \end{bmatrix}$$

- Note that $\mathbb{E}[\mathbf{h}(X_1, X_2)] = [H(X_1|X_2), H(X_2|X_2), H(X_1, X_2)]^T$

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Theorem (T.-Kosut (2012))

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and $\mathcal{S}_{\mathbf{V}}(\epsilon) := \{\mathbf{z} \in \mathbb{R}^3 : \mathbb{P}(\mathbf{Z} \le \mathbf{z}) \ge 1-\epsilon\}$ *with* $\mathbf{Z} \sim \mathcal{N}(\mathbf{0},\mathbf{V})$.

Slepian-Wolf Dispersion: Proof of Direct Part

Direct part:

- Random binning + Joint minimum empirical entropy decoding similar to error exponent analysis in Csiszár and Körner
- Multidimensional Berry-Essèen Theorem

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Theorem (Bentkus 2003)

Let $\mathbf{X}_1, \ldots, \mathbf{X}_n$ be independent and identically distributed random vectors in $\mathbb{R}^d$ with

$$\mathbb{E}(\mathbf{X}_1) = \mathbf{0}_d, \qquad \mathrm{Cov}(\mathbf{X}_1) = \mathbf{I}_d, \qquad \mathbb{E}\|\mathbf{X}_1\|_2^3 = \xi$$

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Then, for every $n \in \mathbb{N}$ *with* $\mathbf{Z} \sim \mathcal{N}(\mathbf{0}_d, \mathbf{I}_d)$,

$$\sup_{\substack{\mathcal{C} \subset \mathbb{R}^d: \\ \mathcal{C} \text{ measurable,convex}}} \left| \mathbb{P}\left(\frac{1}{\sqrt{n}} \sum_{i=1}^n \mathbf{X}_i \in \mathcal{C} \right) - \mathbb{P}(\mathbf{Z} \in \mathcal{C}) \right| \le \frac{400 d^{1/4} \xi}{\sqrt{n}}$$

Slepian-Wolf Dispersion: Proof of Converse Part

Converse part:

- Strong converse in Han's information spectrum book – A result by Miyake and Kanaya (1995)

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$$\epsilon \geq \mathbb{P}\Bigg[-\frac{1}{n}\log p_{X_1^n|X_2^n}(X_1^n|X_2^n) \geq \frac{1}{n}\log M_1 + \gamma$$

$$\text{or } -\frac{1}{n}\log p_{X_2^n|X_1^n}(X_2^n|X_1^n) \geq \frac{1}{n}\log M_2 + \gamma$$

$$\text{or } -\frac{1}{n}\log p_{X_1^n,X_2^n}(X_1^n,X_2^n) \geq \frac{1}{n}\log(M_1M_2) + \gamma \Bigg] - 3(2^{-n\gamma})$$

$$= 1 - \mathbb{P}\left(\mathbf{h}(X_1^n, X_2^n) \leq \mathbf{R} + \gamma\mathbf{1}\right) - 3(2^{-n\gamma}), \qquad \mathbf{R} = [R_1, R_2, R_1 + R_2]^T$$

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- Set $\gamma = O(\frac{\log n}{n})$ and apply the Multidim. Berry-Essèen Theorem

Our Main Result

The (n,ϵ)-optimal rate region for the Slepian-Wolf problem is

$$\begin{bmatrix} R_1 \\ R_2 \\ R_1 + R_2 \end{bmatrix} \in \begin{bmatrix} H(X_1|X_2) \\ H(X_2|X_1) \\ H(X_1,X_2) \end{bmatrix} + \frac{1}{\sqrt{n}} \mathcal{S}_{\mathbf{V}}(\epsilon) \pm O\left(\frac{\log n}{n}\right) \mathbf{1}$$

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What's this special set $\mathcal{S}_{\mathbf{V}}(\epsilon)$?

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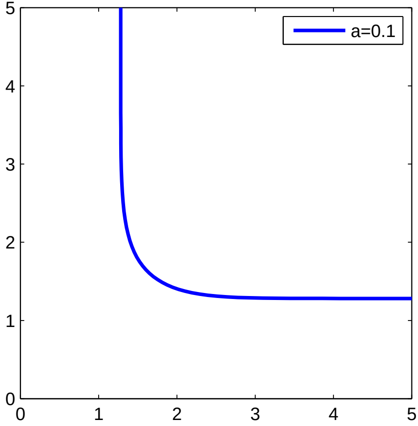

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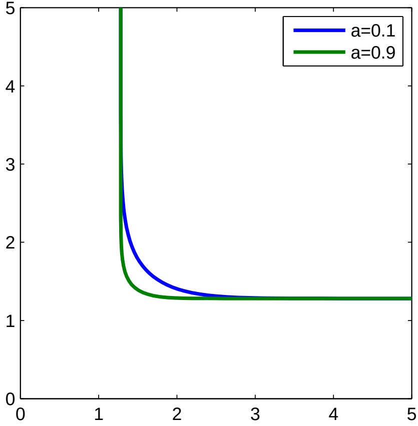

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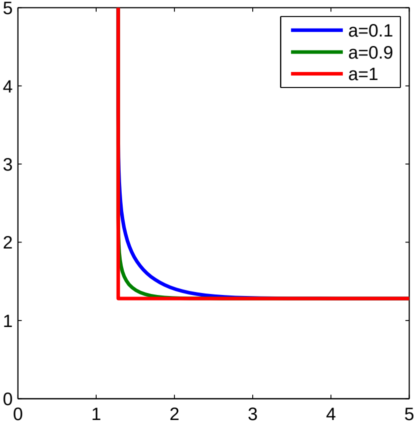

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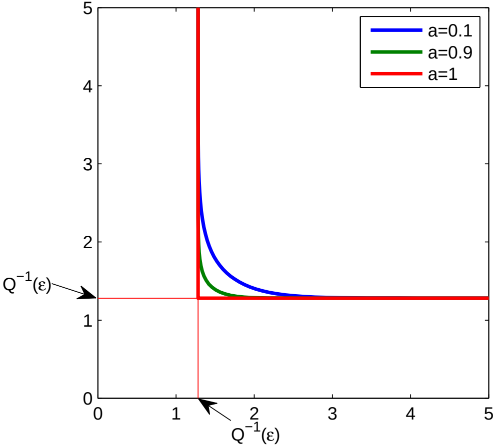
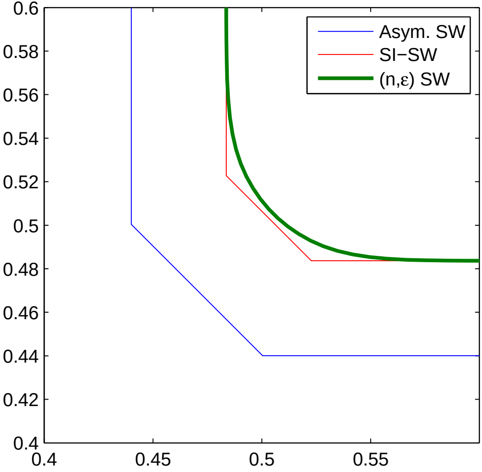


(n, ϵ)-Slepian-Wolf Region

$$p(x_1, x_2) = \begin{bmatrix} 0.7 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

$$\epsilon = 0.1$$

$$n = 300$$



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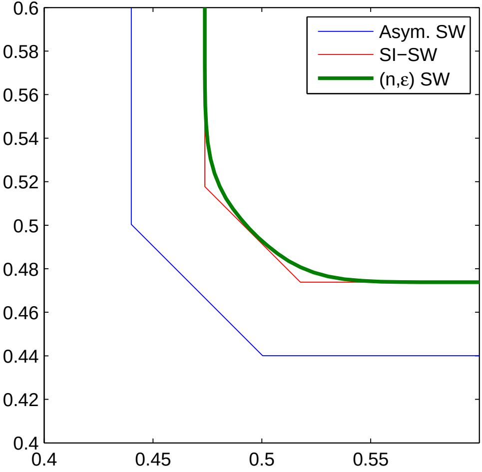
$$p(x_1, x_2) = \begin{bmatrix} 0.7 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

$$\epsilon = 0.1$$

$$n = 500$$



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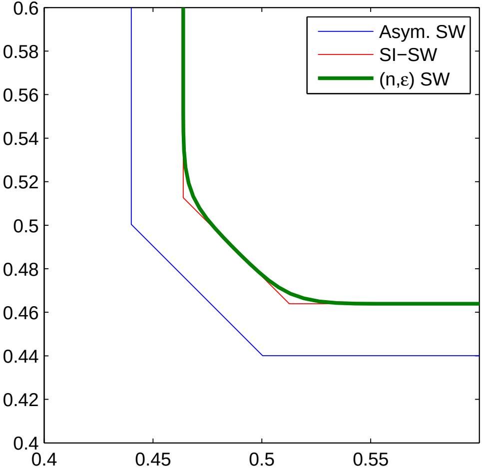

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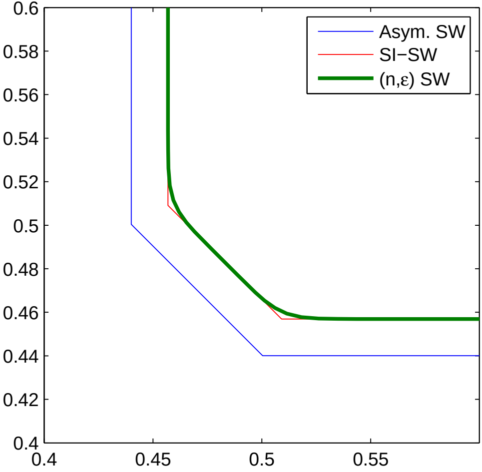
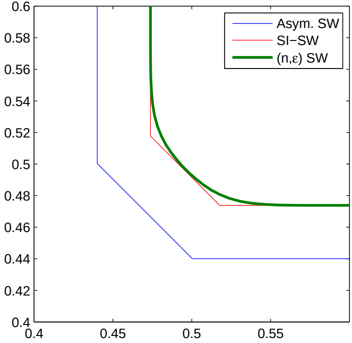
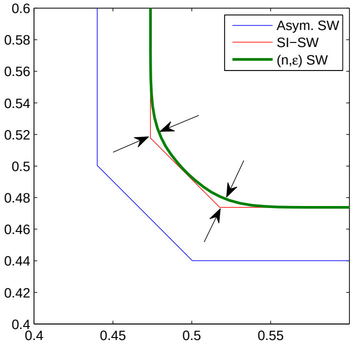


(n, ϵ)-Slepian-Wolf vs (n, ϵ)-side-information SW



Proposition (T.-Kosut (2012))

(n, ϵ)-Slepian-Wolf vs (n, ϵ)-side-information SW



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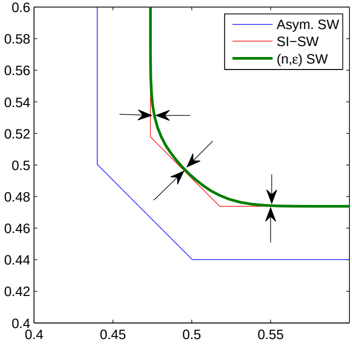

Proposition (T.-Kosut (2012))

- *At corner points, distance is* $\frac{c}{\sqrt{n}}$ *for some* $0 < c < \infty$*.*
- *Away from corner points, distance is* $\exp(-\Theta(n))$

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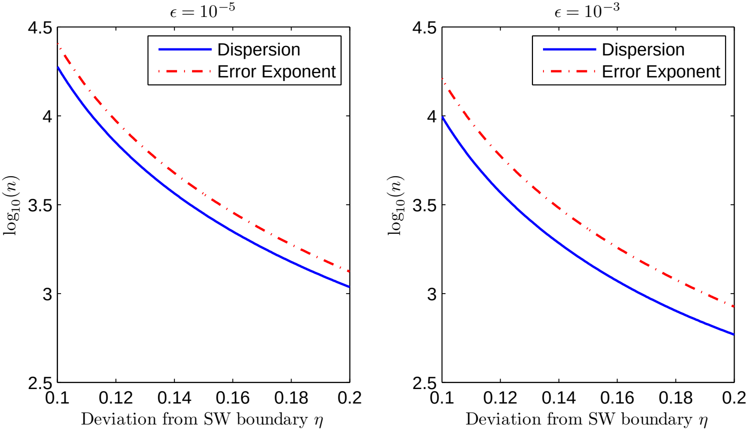
  where $E(R_1, R_2) > 0$ iff (R_1, R_2) is in the interior of the SW region
- Compare the two methods

Dispersion: $$n_{\mathrm{D}} := \min\left\{n : \mathbb{P}(\mathbf{Z} \le \sqrt{n}(\mathbf{R} - \mathbf{H})) \ge 1 - \epsilon\right\}$$

Error Exponent: $$n_{\mathrm{E}} := \left\lceil \frac{1}{E(R_1, R_2)} \log\left(\frac{3}{\epsilon}\right) \right\rceil$$

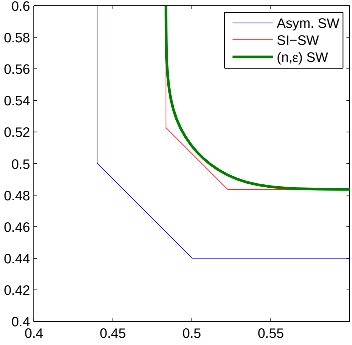
Comparison to Error Exponent Analysis



Prediction of required blocklength n by dispersion analysis (the Gaussian approximation) is much smaller than error exponent analysis!

Summary

- Characterization of (n, ϵ)-Slepian-Wolf rate region up to $O(\frac{\log n}{n})$
- All existing Slepian-Wolf results are corollaries of our theorem
- Required use of dispersion matrix
- Region differs substantively from the SI-SW only at corner points



Extensions

- Achievability technique is exceedingly general
- Can be applied to get inner bounds to the discrete memoryless
  1. Multiple-access channel
  2. Asymmetric broadcast channel (with degraded message sets)
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  4. Transmitting correlated sources over a MAC (Cover-El Gamal-Salehi, 1980)
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- However, other than the Slepian-Wolf problem, it is difficult to use information spectrum methods to get converses for other (n, ϵ)-regions in network information theory.