Wireless Compressive Sensing for Energy Harvesting Sensor Nodes over Fading Channels

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***Abstract*—We consider the scenario in which multiple sensors send spatially correlated data to the fusion center (FC) via independent Rayleigh-fading channels with additive noise. Assuming that the sensor data is sparse in some basis, we show that the recovery of the signal can be formulated as a compressive sensing (CS) problem. To model the scenario where sensors operate with intermittently available energy that is harvested from the environment, we propose that each sensor transmits independently with some probability, and adapts the transmit power to its harvested energy. Due to probabilistic transmissions, the elements of the equivalent sensing matrix are not Gaussian. Since sensors have different energy-harvesting rates and different sensor-to-FC distances, the FC has different receive signal-to-noise ratios (SNRs) for all sensors, referred to as the *inhomogeneity* of SNRs. Thus, the elements of the sensing matrix are also not identically distributed. We provide guarantees on the number of measurements for reliable reconstruction, by showing that the corresponding sensing matrix satisfies the restricted isometry property (RIP), under some mild conditions. We then compute an achievable spectral efficiency (SE) under an allowable mean-square-error (MSE). Furthermore, we analyze the impact of inhomogeneity on the RIP. Our analysis is corroborated by numerical results.**

I. Introduction

Harvesting ambient energy such as solar, wind and thermal energy, has become an appealing solution to prolong the lifetime of wireless sensor networks (WSNs). Energy harvesters provide a virtually perpetual but unreliable energy source, and are potentially maintenance-free [1]. Moreover, the sensors typically have different energy harvesting rates, due to varying harvesting conditions such as spread of sunlight and difference in wind speeds.

This paper addresses the problem of data transmission in energy harvesting WSNs (EHWSNs). We assume that energy harvesting sensors are deployed to monitor some physical phenomenon in space, e.g., temperature, toxicity of gas. Data collected from sensors are sent to the fusion center (FC). The data are typically correlated, and well approximated by a sparse vector in an appropriate transform (e.g., the Fourier Transform). Recent developments in compressive sensing (CS) theory provide efficient methods to recover sparse signals from limited measurements [2]. CS theory states that if the sensing matrix satisfies the restricted isometry property (RIP), a small number of measurements (relative to the length of the data vector) is sufficient to accurately recover the sparse data. This advantage of CS potentially allows us to reduce the total number of transmissions and hence increase the spectral efficiency (SE).

The estimation of the sensor data accurately by the FC has recently been addressed by using CS techniques in the literature. In [3], Haupt *et. al* presented a sensing scheme based on phase-coherent transmissions for all sensors. However, [3] made two impractical assumptions. First, it assumed that there was no channel fading, and path losses for all sensors were identical; Second, the transmissions from all sensors were synchronized such that signals arrived in phase at the FC. In [4], Aeron *et. al* derived information theoretic bounds on sensing capacity of sensor networks under fixed signal-to-noise ratio (SNR). In contrast, [5] proposed a sparse approximation method in non-fading channels, which adapted a sensor's sensing activity according to its energy availability. In [6], Fazel *et. al* proposed a random access scheme in underwater sensor networks. Each activated sensor picked a uniformly-distributed delay to transmit. By simply discarding the colliding data packets from concurrent medium access, the FC used a CS decoder to recover the sensor data based only on the successfully received packets. Thus, the scheme did not exploit packet collisions for data recovery.

Since sensors are placed at different locations, it is commonly assumed that the sensors transmit data over independent but nonidentical channels with different fading conditions. Different energy-harvesting rates also lead to different transmit powers and hence different (receive) SNRs. We refer to this generally as the *inhomogeneity* of EHWSNs. The application of *wireless compressive sensing* to inhomogeneous EHWSNs has, to the best of our knowledge, not been studied in literature. We aim to reduce the number of transmissions and thus to improve the SE. The three main contributions are summarized as follows.

First, we present an efficient probabilistic transmission scheme that essentially performs CS over the air. Every sensor transmits with some probability, and adjusts the transmit power according to its energy availability. The FC thus receives a linear combination of the transmitted signals with some additive noise.

Second, we prove that the FC can recover the data accurately, if the total number of transmissions (or measurements) m exceeds

$$O\left(k\frac{\rho_{\max}(k)}{\rho_{\min}(k)}\log\frac{n}{k}\right),$$

where n is the number of sensors, k is the sparsity of the sensor data, and $\rho_{\max}(k)$ and $\rho_{\min}(k)$ are respectively the maximum and minimum k-restricted eigenvalues of a Gram matrix which depends on the inhomogeneity of the SNRs.

Third, we analyze the impact of inhomogeneity on the required number of measurements, in terms of $\rho_{\max}(k)$ and $\rho_{\min}(k)$. We model that the signal powers of the sensors follow independent truncated Gaussian distributions. By using the theory of large deviations, we show that both $\rho_{\max}(k)$ and $\rho_{\min}(k)$ concentrate around one (for all k) in large n region, and the rate of convergence to one depends on the inhomogeneity.

II. System Model

In the WSN under consideration, n energy harvesting sensors transmit to the FC via a shared multiple-access channel (MAC). We consider slotted transmissions, and m successive time slots constitute a frame. We consider a snapshot of the spatial-temporal field, and transmissions within one frame. Assuming the sensor data $\mathbf{s}$ is compressible, we can model it as being sparse w.r.t. to a fixed orthonormal basis $\{\boldsymbol{\psi}_j \in \mathbb{C}^n : j = 1, \ldots, n\}$, i.e.,

$$\mathbf{s} = \boldsymbol{\Psi}\mathbf{x} = \sum_{j=1}^{n} \boldsymbol{\psi}_j x_j, \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^n$ has at most $k < \lfloor n/2 \rfloor$ nonzero components, where $\lfloor\cdot\rfloor$ is the floor operation.

We assume a flat-fading channel with complex-valued channel coefficients h_{ij}, where $1 \le i \le m$ denotes the slot index and $1 \le j \le n$ denotes the sensor index. The channel remains constant in each slot. We further assume a Rayleigh fading channel, hence the channel coefficients for different slots are independent and identically distributed (i.i.d.) according to the complex Gaussian distribution.

We propose simultaneous transmissions to the FC such that data from all sensors are linearly combined. Sensor j multiplies its observation s_j by some random amplitude ϕ_{ij} (to be defined in (3)), then transmits in the i-th time slot. The FC thus receives

$$y_i = \sum_{j=1}^{n} h_{ij}\phi_{ij}s_j + e_i,$$

where e_i is a noise term (not necessarily Gaussian). After m time slots, the FC receives the measurement vector

$$\mathbf{y} = (\mathbf{H} \odot \boldsymbol{\Phi})\mathbf{s} + \mathbf{e} = \mathbf{Z}\mathbf{s} + \mathbf{e} = \mathbf{Z}\boldsymbol{\Psi}\mathbf{x} + \mathbf{e}, \tag{2}$$

where the matrix $\mathbf{Z} = \mathbf{H} \odot \boldsymbol{\Phi}$, and the operation $\odot$ is the element-wise product of two matrices. We assume all noise components are independent, with zero mean and variance σ^2. With the knowledge of the sensing matrix $\mathbf{Z}$ and the sparsity-inducing basis $\boldsymbol{\Psi}$, the FC can implement CS decoding to recover sparse coefficients $\widehat{\mathbf{x}}$ and obtain the estimated data vector $\widehat{\mathbf{s}} = \boldsymbol{\Psi}\widehat{\mathbf{x}}$.

We want to estimate $\mathbf{x}$ or equivalently $\mathbf{s}$, from $\mathbf{y}$, such that the *mean square error* (MSE) $\epsilon \triangleq \mathbb{E}\|\widehat{\mathbf{x}} - \mathbf{x}\|_2^2$ does not exceed some threshold. Given a fixed number of sensors n and the MSE threshold, our objective is to design a transmission scheme that minimizes the number of transmissions m, and thus achieves high SE.

A. Energy-Aware Wireless Compressive Sensing

We consider only the energy consumption for wireless transmissions. The energy harvesting rate varies over sensors. For simplicity, we assume that each sensor allocates the same power for all slots of one frame. Define the *energy level* as the amount of available energy of a sensor in one slot. The energy level of sensor j is denoted as b_j, the unit of which is joule per slot. We use a fraction $p \in (0, 1]$ of energy level to transmit in one frame, and the saved energy can be used in future frames.

We perform energy-aware wireless transmissions taking into account the different harvested energy rates. Based on the aforementioned energy allocation, we design $\boldsymbol{\Phi}$ in (2) as a *select-and-weight* (SW) matrix, whose elements are independently generated according to the following probability mass function

$$\phi_{ij} = \begin{cases} +\sqrt{b_j} & \text{w.p. } p/2 \\ 0 & \text{w.p. } 1-p, \quad \forall i = 1, \ldots, m \\ -\sqrt{b_j} & \text{w.p. } p/2 \end{cases} \tag{3}$$

That is, the j-th sensor transmits with probability p with an amplitude of $\sqrt{b_j}$, and the actual value is positive or negative with equal probability.

B. Probability Distribution and Signal Model

Consider the signal model in (2). Denote each element in $\mathbf{Z}$ as $Z_{ij} = h_{ij}\phi_{ij} = Z_{ij}^{\mathrm{R}} + jZ_{ij}^{\mathrm{I}}$, where $Z_{ij}^{R} \triangleq h_{ij}^{\mathrm{R}}\phi_{ij}$, and $Z_{ij}^{\mathrm{I}} \triangleq h_{ij}^{\mathrm{I}}\phi_{ij}$. Since all elements of matrix $\mathbf{H}$ are assumed to be independent, all elements of matrix $\mathbf{Z}$ are independent, and with independent real and imaginary components. Thus, it suffices to analyze the probability density function (pdf) of the real component.

Definition 1. *A random variable X follows the* mixed Gaussian *distributions, denoted as $X \sim \widetilde{\mathcal{N}}(\mu, \nu^2, p)$, if* *it has the following pdf*

$$f_X(x) = p\frac{1}{\sqrt{2\pi\nu^2}}\exp\left(-\frac{(x-\mu)^2}{2\nu^2}\right)+(1-p)\delta(x), \quad (4)$$

where the parameter $p \in (0,1]$. *The complex mixed Gaussian distribution, denoted as* $\widetilde{\mathcal{N}}_c(\mu,\nu^2,p)$, *has independent real and imaginary components that are distributed as* $\widetilde{\mathcal{N}}(\mu,\nu^2/2,p)$.

All elements in the channel matrix $\mathbf{H}$ are independent zero mean, Gaussian random variables. Note that $\mathbf{H}$ has column-dependent variances ν_j^2, due to different fading channels for the sensors. Clearly, $Z_{ij}^{\mathrm{R}} \sim \widetilde{\mathcal{N}}\big(0,\nu_j^2 b_j/2,p\big)$.

Let $\mathbf{H} = \widetilde{\mathbf{H}}\mathbf{\Gamma}_{\mathrm{H}}$ and $\mathbf{\Phi} = \widetilde{\mathbf{\Phi}}\mathbf{\Gamma}_{\Phi}$, where $\mathbf{\Gamma}_{\mathrm{H}} = \mathrm{diag}\{\nu_1,\nu_2,\ldots,\ \nu_n\}$ and $\mathbf{\Gamma}_{\Phi} = \mathrm{diag}\{\sqrt{pb_1},\sqrt{pb_2},\ldots,\sqrt{pb_n}\}$. Then we decompose the matrix $\widetilde{\mathbf{Z}}$ as follows

$$\mathbf{Z} = \sqrt{m}\widetilde{\mathbf{Z}}\mathbf{\Gamma}, \quad (5)$$

where we denote $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{H}} \odot \widetilde{\mathbf{\Phi}}$ and $\mathbf{\Gamma} = \mathbf{\Gamma}_{\mathrm{H}}\mathbf{\Gamma}_{\Phi}$. Let $\mathbf{\Gamma} = \mathrm{diag}\{\sqrt{\gamma_1},\sqrt{\gamma_2},\ldots,\sqrt{\gamma_n}\}$, where the receive signal power of sensor j is $\gamma_j = pb_j\nu_j^2$. The γ_j's are generally all different, and this is referred to as an *inhomogeneous signal-power pattern*. We note that all elements of the matrix $\widetilde{\mathbf{Z}}$ are i.i.d. mixed Gaussian random variables, i.e., $\widetilde{Z} \sim \widetilde{\mathcal{N}}_c\left(0,1/(pm),p\right)$ and $\widetilde{Z}^{\mathrm{R}} \sim \widetilde{\mathcal{N}}\left(0,1/(2pm),p\right)$.

Using (5), we rewrite the signal model in (2) as

$$\mathbf{y} = \sqrt{m}\widetilde{\mathbf{Z}}\mathbf{\Gamma}\mathbf{\Psi}\mathbf{x} + \mathbf{e}. \quad (6)$$

We rescale the matrix $\mathbf{\Gamma}\mathbf{\Psi}$ such that each column has unit Euclidean norm. Let $\mathbf{\Sigma} = \mathbf{\Gamma}\mathbf{\Psi}/\sqrt{\mathrm{P}_{\mathrm{ave}}}$, where $\mathrm{P}_{\mathrm{ave}} = \sum_{j=1}^n pb_j\nu_j^2/n$ denotes the average (receive) signal power in one time slot. By dividing both sides of (6) by $\sqrt{m\mathrm{P}_{\mathrm{ave}}}$, we thus obtain the signal model

$$\widetilde{\mathbf{y}} = \widetilde{\mathbf{Z}}\mathbf{\Sigma}\mathbf{x} + \widetilde{\mathbf{e}} = \mathbf{A}\mathbf{x} + \widetilde{\mathbf{e}}, \quad (7)$$

where all noise components are independent, with zero mean and normalized variance $\widetilde{\sigma}^2 = \sigma^2/(m\mathrm{P}_{\mathrm{ave}})$. The average (receive) SNR is defined as $\mathrm{SNR}_{\mathrm{ave}} \triangleq \frac{P_{\mathrm{ave}}}{\sigma^2}$.

III. Main Analytical Results

We recall the definition of RIP [7] and state our main result, that is Theorem 1, in subsection III-A. The engineering implication of Theorem 1, and in particular the tradeoff between the allowable MSE and the achievable SE, will be discussed in III-B. Finally we analyze the effect of inhomogeneity of SNRs in subsection III-C.

A. Restricted Isometry Property

It is well-established in CS theory that a sufficient condition for accurate and efficient reconstruction (via convex optimization) is that the sensing matrix satisfies the RIP. A matrix $\mathbf{A}$ is said to satisfy RIP of order k, if there exists a $\delta_k \in (0,1)$ such that

$$(1-\delta_k)\|\mathbf{x}\|_2^2 \le \|\mathbf{A}\mathbf{x}\|_2^2 \le (1+\delta_k)\|\mathbf{x}\|_2^2 \quad (8)$$

holds for all k-sparse vectors. The smallest constant satisfying (8) is the restricted isometry constant (RIC) [7]. Hence, we establish conditions to show when the sensing matrix $\mathbf{A}$ obeys the RIP.

We define the *k-restricted extreme eigenvalues* of the Gram matrix $\mathbf{\Sigma}^*\mathbf{\Sigma}$ as

$$\rho_{\max}(k) = \max_{\mathbf{v}:\|\mathbf{v}\|_0\le k,\|\mathbf{v}\|_2=1}\|\mathbf{\Sigma}\mathbf{v}\|_2^2,$$
$$\rho_{\min}(k) = \min_{\mathbf{v}:\|\mathbf{v}\|_0\le k,\|\mathbf{v}\|_2=1}\|\mathbf{\Sigma}\mathbf{v}\|_2^2, \quad (9)$$

where $\mathbf{v} \in \mathbb{C}^n$, and the "$l_0$-norm" $\|\mathbf{v}\|_0$ refers to the number of non-zero elements of $\mathbf{v}$. Clearly, $\rho_{\max}(1) = \rho_{\min}(1) = 1$. The extreme eigenvalues will be used to understand how the inhomogeneous SNRs affects the RIP. It can be easily checked that the following bounds hold [8]:

$$1 \le \rho_{\max}(k) \le k, \qquad 0 \le \rho_{\min}(k) \le 1. \quad (10)$$

We note that k typically satisfies $k \ll n$ in large-scale WSNs. We further assume $\rho_{\max}(k) \in [1,2]$ to simplify some of the mathematical arguments. We show that this assumption holds with high probability in Section III-C. To state our main result clearly, we define two quantities that depend on $\mathbf{\Sigma}$ and k as follows

$$\xi_k(\mathbf{\Sigma}) \triangleq \max\left\{1-\rho_{\min}(k),\rho_{\max}(k)-1\right\},$$
$$\zeta_k(\mathbf{\Sigma}) \triangleq \max\left\{0,\frac{2-\rho_{\max}(k)-\rho_{\min}(k)}{\rho_{\max}(k)-\rho_{\min}(k)}\right\}. \quad (11)$$

Since $\rho_{\max}(k) \in [1,2]$, we have[1] $\xi_k,\zeta_k \in [0,1]$. Let $\vartheta_k = (1+\zeta_k)\rho_{\max}(k)-1$. Given $\delta_k \in (\xi_k,1)$, for convenience, we map δ_k to a "modified RIC" via a piecewise linear mapping as follows

$$\beta_k(\delta_k,\mathbf{\Sigma}) \triangleq \begin{cases} 1-(1-\delta_k)/\rho_{\min}(k), & \delta_k \in \big(\xi_k,\vartheta_k\big) \\ (1+\delta_k)/\rho_{\max}(k)-1, & \delta_k \in \big(\vartheta_k,1\big). \end{cases} \quad (12)$$

Let $\varsigma_k = 2/\rho_{\max}(k)-1$. The inverse of $\beta_k(\delta_k,\mathbf{\Sigma})$ is denoted as

$$\delta_k(\beta_k,\mathbf{\Sigma}) \triangleq \begin{cases} 1-(1-\beta_k)\rho_{\min}(k), & \beta_k \in \big(0,\zeta_k\big) \\ (1+\beta_k)\rho_{\max}(k)-1, & \beta_k \in \big(\zeta_k,\varsigma_k\big). \end{cases} \quad (13)$$

Recall that the sensing matrix $\mathbf{A} = \widetilde{\mathbf{Z}}\mathbf{\Sigma}$, where all elements of the $m \times n$ matrix $\widetilde{\mathbf{Z}}$ are i.i.d. mixed Gaussian random variables, where n is the number of sensors. We state the main result as follows.

Theorem 1. *Let* $c_1, c_2 > 0$ *be some universal constants. Given a sparsity level* $k < \lfloor n/2 \rfloor$, *a transmit probability* $p \in (0,1]$ *and a number* $\delta_k \in (\xi_k,1)$, *if the number of measurements satisfies*

$$m > \frac{c_1 k\rho_{\max}(k)}{p^2\beta_k^2\rho_{\min}(k)}\log\frac{5en}{k}, \quad (14)$$

[1] Note that the arguments of some quantities are sometimes omitted.

where $\beta_k = \beta_k(\delta_k, \mathbf{\Sigma})$ is defined in (12)*, then for any vector* $\mathbf{x}$ *with support of cardinality of at most k, we have that the RIP in* (8) *holds with probability at least* $1 - \exp\left(-c_2 m p^2 \beta_k^2/4\right)$.

Proof: (Sketch) We show that the rows of the random matrix $\widetilde{\mathbf{Z}}$ are isotropic sub-Gaussian [9], and $\mathbf{A}$ is an approximate isometry when restricted to the set of sparse vectors. See details in [8]. ■

Remark 1 (Specialization to the homogeneous case). Clearly, the lower bound on the required number of measurements is $O(\frac{k\rho_{\max}(k)}{\beta_k^2 \rho_{\min}(k)} \log \frac{n}{k})$. For the homogeneous signal-power pattern (i.e., the matrix $\mathbf{\Gamma}$ is a multiple of the identity matrix), we have $\rho_{\max}(k) = \rho_{\min}(k) = 1$ and $\beta_k = \delta_k$. Thus the lower bound reduces to $O(\frac{k}{\delta_k^2} \log \frac{n}{k})$, which coincides with the known results for i.i.d. random sensing matrices. See Theorem 5.2 in [10] and Section 1.4.4 in [9].

Remark 2 (Contribution of the RIP analysis). Due to the inhomogeneous signal-power pattern $\mathbf{\Gamma}$, the rows $\mathbf{a}_i$ of the sub-Gaussian sensing matrix $\mathbf{A}$ are *non-isotropic*. To the best of our knowledge, little is known about the RIP of non-isotropic sub-Gaussian random matrices. The only relevant result is in Remark 5.40 in [9] which gives a concentration inequality of non-isotropic random sensing matrices in terms of the upper bound on the spectral norm. However, the authors did not demonstrate how the inhomogeneity affects the RIP, nor they investigate the number of measurements required to satisfy the RIP. Theorem 1 fills this gap.

B. Achievable Spectral Efficiency

The SE is defined as the ratio[2] $\eta \triangleq n/m$. It is interpreted as the dimension of spatial-field signal vector that is reliably recovered at the FC in one time slot via wireless transmissions. Given the number of sensors n and an allowable MSE level $\epsilon > 0$, an achievable (highest) SE is defined as

$$\eta(\epsilon) \triangleq \max_m \frac{n}{m} \quad \text{subject to} \quad \mathbb{E}\|\widehat{\mathbf{x}} - \mathbf{x}\|_2^2 \leq \epsilon. \tag{15}$$

Corollary 1. *Let $p, m, n, k, \mathbf{\Sigma}, \xi_k, \vartheta_k$ be as in Theorem 1. Let $\epsilon_{\text{th}} = 1/(0.0942 \times \text{SNR}_{\text{ave}})$. Given an allowable MSE level $\epsilon > \epsilon_{\text{th}}$, with overwhelming probability, the achievable SE is*

$$\eta(\epsilon) = \frac{np^2(\tilde{\beta}_k)^2 \rho_{\min}(k)}{c_1 k \rho_{\max}(k) \log \frac{5en}{k}}, \tag{16}$$

where

$$\tilde{\beta}_k(\mathbf{\Sigma}, \epsilon) = \begin{cases} 1 - \dfrac{0.693 + 1/\sqrt{\epsilon \text{SNR}_{\text{ave}}}}{\rho_{\min}(k)}, \ \delta_k \in \left(\xi_k, \vartheta_k\right), \\ \dfrac{1.307 - 1/\sqrt{\epsilon \text{SNR}_{\text{ave}}}}{\rho_{\max}(k)} - 1, \ \delta_k \in \left(\vartheta_k, 1\right). \end{cases} \tag{17}$$

Proof: (Sketch) The proof is based on Theorem 3.2 of [11], which states that if $\mathbf{A}$ satisfies RIP of order k with RIC $\delta_k < 0.307$, the MSE

$$\mathbb{E}\|\widehat{\mathbf{x}} - \mathbf{x}\|_2^2 \leq \frac{1}{\text{SNR}_{\text{ave}}(0.307 - \delta_k)^2}. \tag{18}$$

A desired MSE level requires a sufficient small RIC. Then applying our Theorem 1 establishes the result. See further details in [8]. ■

Remark 3. Note that Corollary 1 applies only to the case where the MSE is greater than the MSE threshold ϵ_{th}. If $\epsilon \leq \epsilon_{\text{th}}$, then from (18), simple algebra reveals that $\delta_k = 0$, which implies that the sensing matrix $\mathbf{A}$ is a perfect isometry. Since $\mathbf{A}$ is random, and the entries are governed by a distribution that is absolutely continuous w.r.t. the Lebesgue measure, this occurs with probability zero, implying that the constraint in (15) is never satisfied. Thus, we define the SE in this case to be zero.

Remark 4. We note that $\tilde{\beta}_k$ is an increasing function of ϵ and SNR_{ave}, and so is the SE $\eta(\epsilon)$. Another important insight we can draw from Corollary 1 is that the key measure for the inhomogeneity of SNRs is the ratio $r \triangleq \rho_{\max}(k)/\rho_{\min}(k)$. The SE decreases as r increase from one. We hence analyze the impact of inhomogeneity of SNRs on the deviation of $\rho_{\max}(k)$ and $\rho_{\min}(k)$ from unity in subsection III-C. In addition, note that the SE in (16) decrease quadratically as the transmit probability p decreases. Thus, it is always advantageous to transmit with high probability subject to energy constraint.

Example 1. Let number of sensors $n = 500$, sparsity level $k = 5$ and transmit probability $p = 0.8$. These parameters imply $\rho_{\max}(k) = 1.09, \rho_{\min}(k) = 0.88$ (See Section IV). We plot the achievable SE against an allowable MSE, under different average SNR settings in Fig. 1. We observe that beyond the MSE threshold that depends on the average SNR, the achievable SE increases as either the allowable MSE or the average SNR increases, which is is expected.

C. Impact of Inhomogeneity

This subsection investigates the impact of inhomogeneity of receive signal powers, or equivalently the inhomogeneity of SNRs (assuming all sensors have the same noise power), on the RIP. We focus on the asymptotic scenario where the number of sensors n tends to infinity. To make the dependence on n clear, we denote $\rho_{\max}(k)$ as $\rho_{\max}(k, n)$, and $\rho_{\min}(k)$ as $\rho_{\min}(k, n)$. Recall their

[2]The usage of the term SE in (15) deviates from the usual conventions in digital communications. However, our motivation is as follows: Each sensor encodes its observation into D-bit data, and then transmits to the FC in a time slot of T_0 seconds. After transmissions using bandwidth $B = D/T_0$ in m slots, the FC reliably recovers nD bits data in mT_0 seconds. Thus the SE is derived as $\eta = nD/(mT_0 B) = n/m$.

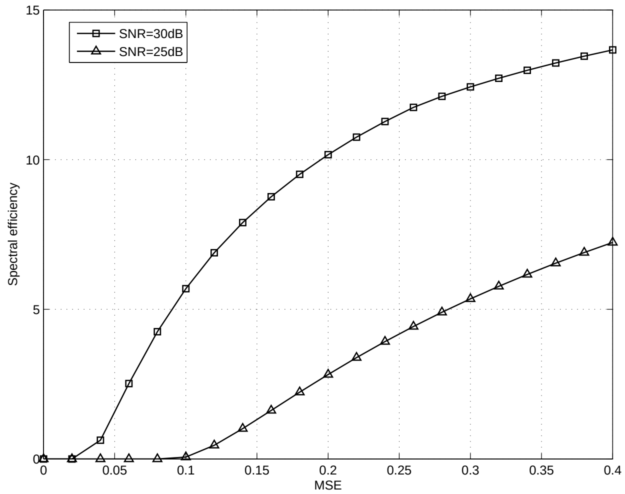

Fig. 1. Spectral efficiency vs. allowable MSE.

definitions in (9) and the signal model in (7). Let $\mathbf{w} = \mathbf{\Sigma v}$, where the unit-norm, k-sparse vector $\mathbf{v}$ is supported on the set $\mathcal{T} \triangleq \{s_1, \ldots, s_k\}$. Without loss of generality, let $s_1 < \cdots < s_k$. To obtain further insights, we take matrix $\mathbf{\Psi}$ as the n-point discrete Fourier matrix (We note that Theorem 2 also applies for a larger class of sparsity-inducing bases. See Remark 6 for a more in-depth discussion on this point.). Hence, we get

$$\|\mathbf{w}\|_2^2 = 1 + \sum_{i=1}^{n} \frac{\gamma_i}{nP_{\text{ave}}} \left(1 + \sum_{q=1}^{k} \sum_{t=1, t<q}^{k} 2\text{Re}\left\{ v_{s_q} v_{s_t}^* \exp\left(\frac{-j2\pi(i-1)(s_q - s_t)}{n}\right)\right\}\right). \tag{19}$$

We are interested to know how $\rho_{\max}(k,n)$ and $\rho_{\min}(k,n)$ vary with different receive signal powers. Thus, we consider a model in which the signal powers γ_i's are i.i.d. random variables following approximately a Gaussian distribution. By varying the variance of the Gaussian distribution, we are in fact varying the inhomogeneity of the receive signal powers. Specifically, to deal with the fact that the signal powers can not be negative, we use the following truncated Gaussian distribution to model the receive signal powers.

Definition 2. *A random variable X is truncated Gaussian, denoted as $\mathcal{N}_{\text{tr}}(\mu, \omega^2)$, if its pdf is*

$$g_X(x; \mu, \omega) = \frac{1}{\sqrt{2\pi}\omega(1 - Q(\mu/\omega))} \exp\left(-\frac{(x-\mu)^2}{2\omega^2}\right), \tag{20}$$

for $x \geq 0$ and 0 else, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$ is the Q-function of a standard Gaussian function.

We assume that $\gamma_i \sim \mathcal{N}_{\text{tr}}(\mu, \omega^2)$ for all $i = 1, \ldots, n$. Given μ, the "variance" ω^2 is a measure of the degree of inhomogeneity of the signal powers γ_i. We use the notation $a_n \dot{\leq} \exp(-nE)$ to mean that $\limsup_{n\to\infty} \frac{1}{n} \log a_n \leq -E$. Under the above assumptions on the statistics of the signal power, we have the following large deviations upper bound on $\rho_{\max}(k,n)$ and $\rho_{\min}(k,n)$:

Theorem 2. *For any $t > 0$, and $1 \leq k < \lfloor n/2 \rfloor$,*

$$\begin{aligned} \mathbb{P}\left(\rho_{\max}(k,n) > 1 + t\right) &\dot{\leq} \exp\left[-nd^2 E(k,t)^2\right], \\ \mathbb{P}\left(\rho_{\min}(k,n) < 1 - t\right) &\dot{\leq} \exp\left[-nd^2 E(k,t)^2\right], \end{aligned} \tag{21}$$

where $E(k,t) \triangleq t/(k - 1 + \sqrt{2}t)$, and $d \triangleq \mu/\omega$.

Proof: (Sketch) We bound the inner sum in (19) using the theory of large deviations [12]. See [8] for details. ■

Remark 5. We note that $E(k,t)$ is an increasing function of t and a decreasing function of the sparsity k which is expected. Also, the exponent $d^2E(k,t)^2$ increases with d, which means that the convergence of $\rho_{\max}(k,n)$ and $\rho_{\min}(k,n)$ to unity is faster when d is large, or equivalently, when the receive signal powers are more homogeneous. It is observed that $\rho_{\max}(k,n)$ approximates one in large n region. This validates the assumption that $\rho_{\max}(k,n) \in [1,2]$ in section III-A.

Remark 6. The only property of the discrete Fourier transform that we exploit in the proof of Theorem 2 is its circular symmetry, i.e., each basis vector (containing elements that are powers of the n-th root of unity) is uniformly distributed over the circle in the complex plane. Hence, certain Cesaro sums converge to zero and the proof goes through. Thus, Theorem 2 also applies for other sparsity-inducing bases whose basis vectors are circularly symmetric. See details in [8].

IV. Simulation Results

We now illustrate our results via numerical examples. We set the number of sensors $n = 500$ and transmit probability $p = 0.8$. We use the truncated Gaussian distribution with $\mu = 0.2$ and $d = 2$, implying $\omega = \mu/d = 0.1$, to model the receive signal powers, and use the basis pursuit de-noising (BPDN) algorithm [13] as the CS decoder.

Fig. 2 plots the MSE vs. the number of measurements (or transmissions) m for different sparsity k and different average SNRs. As expected, the MSE decreases as k decreases, and the average SNR increases. We note that when the average SNR is 25 dB, for an MSE level 2×10^{-3}, the scheme achieves a higher SE of $\eta = 7$ for $k = 5$ compared to $\eta = 4.5$ for $k = 10$.

Fig. 3 shows the cumulative distribution function (CDF) of $\rho_{\max}(k,500)$ and $\rho_{\min}(k,500)$ for $\mu = 0.2$ and $d = 1,2$. Both $\rho_{\max}(k,500)$ and $\rho_{\min}(k,500)$ converge to one faster for large d, or equivalently, for more homogeneous signal powers.

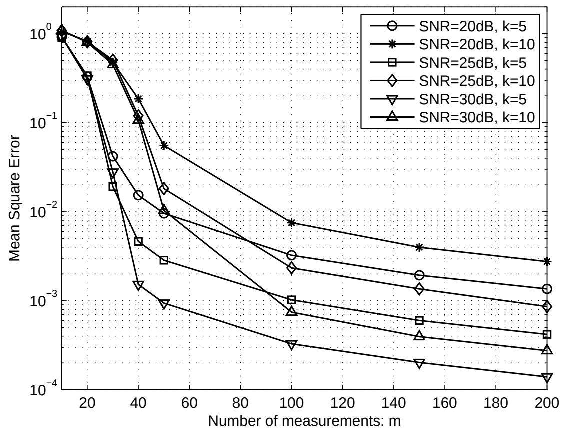

Fig. 2. MSE v.s. number of measurements (m)

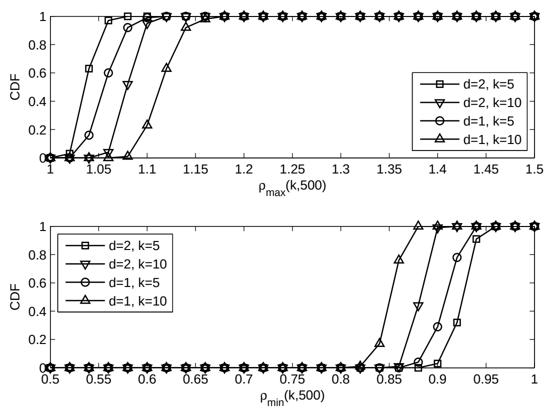

Fig. 3. CDF of $\rho_{\max}(k, 500)$ and $\rho_{\min}(k, 500)$

Finally, we numerically validate the asymptotic behavior of $\rho_{\max}(k, n)$ as n grows. Set $k = 5$, $d = 1, 2, 3$. Fig. 4 shows the probability that $\rho_{\max}(k, n) > 1.04$ for different n. It is observed that the logarithm of the probability decreases linearly as n grows (when n/k is large) and furthermore, the slope varies quadratically w.r.t. d. Thus, our numerical results corroborate the theoretical result in Theorem 2.

V. CONCLUSION

In this paper, to achieve high spectral efficiency in EHWSNs, we propose that each sensor independently decides to transmit or not with some probability, and the transmission power depends on its available energy level. Hence, only a subset of sensors transmit simultaneously to the FC, and the concurrent transmission exploits the spatial combination inherent in a multiple-access channel. We use techniques from CS theory to prove a lower bound on the required number of measurements to satisfy

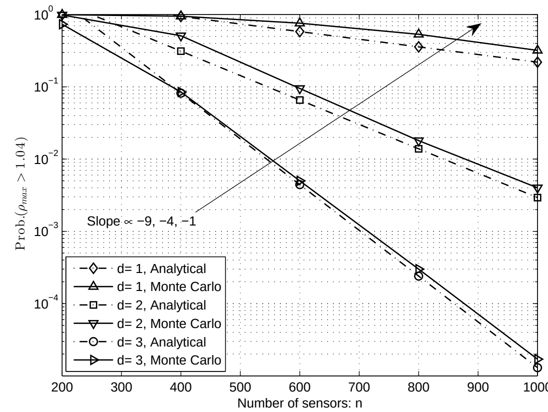

Fig. 4. The prob. of $\rho_{\max}(k, n) > 1.04$ v.s. number of sensors n

the RIP and hence to ensure that the reconstruction is both computationally efficient (and amenable to convex optimization techniques) and accurate. We also derive an achievable spectral efficiency given an allowable MSE. Finally, we analyze the impact of inhomogeneity on the k-restricted extreme eigenvalues. These eigenvalues govern the number of measurements required for the RIP to hold.

REFERENCES

[1] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints", *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4808-4818, Sep. 2012.

[2] E. J. Candes and M. B. Wakin, "An introduction to compressive sampling", *IEEE Signal Process. Mag.*, pp. 21-30, Mar. 2008.

[3] J. D. Haupt and R. D. Nowak, "Signal reconstruction from noisy random projections", *IEEE Trans. Inf. Theory*, vol. 52, pp. 4036-4048, Sep. 2006.

[4] S. Aeron, M. Zhao, and V. Saligrama, "Information theoretic bounds to sensing capacity of sensor networks under fixed SNR", *IEEE Inf. Theory Workshop*, Lake Tahoe, CA, USA, Sep. 2007.

[5] R. Rana, W. Hu, and C. T. Chou, "Energy-aware sparse approximation technique (EAST) for rechargeable wireless sensor networks", in *Proc. of Europ. Conf. on Wireless Sensor Networks*, pp. 306-321, Coimbra, Portugal, Feb. 2010.

[6] F. Fazel, M. Fazel, and M. Stojanovic, "Random access compressed sensing for energy-efficient underwater sensor networks", *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp.1660-1670, Sep. 2011.

[7] E. J. Candes and T. Tao, "Decoding by linear programming", *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4203-4215, Dec. 2005.

[8] G. Yang, V. Y. F. Tan, C. K. Ho, S. H. Ting, and Y. L. Guan, "Wireless compressive sensing for energy harvesting sensor nodes", *in preparation*.

[9] Y. C. Eldar and G. Kutyniok, Compressed sensing: theory and applications, Cambridge University Press, May 2012.

[10] R. Baraniuk, M. Davenport, R. D. Vore, and M. Wakin, "A simple proof of the restricted isometry property", *Constructive Approximation*, vol. 28, no. 3, pp. 253-263, 2008.

[11] T. T. Cai, M. Wang, and G. Xu, "New bounds for restricted isometry constants", *IEEE Trans. Inf. Theory*, vol. 56, no. 9, pp. 4388-4394, Sep. 2010.

[12] A. Dembo and O. Zeitouni, Large deviation techniques and applications, 2nd ed. New York: Springer-Verlag Press, 1998.

[13] E. V. D. Berg and M. P. Friedlander, "Probing the pareto frontier for basis pursuit solutions", in *Proc. of Society for Industrial and Applied Mathematics*, vol. 31, no. 2, pp. 890-912, 2008.