Transmission of Correlated Sources over a MAC: A Gaussian Approximation-Based Analysis

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Transmission of Correlated Sources over a MAC

- We have a 2-DMS S^n, T^n with distribution

$$p_{S^n,T^n}(s^n, t^n) = \prod_{i=1}^{n} p_{S,T}(s_i, t_i)$$

- They are to be separately encoded so $x_1^n = \phi_n^1(s^n) \in \mathcal{X}_1^n$ and $x_2^n = \phi_n^2(t^n) \in \mathcal{X}_2^n$

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- Decoder receives y^n and estimates $\hat{s}^n$ and $\hat{t}^n$, i.e., $(\hat{s}^n, \hat{t}^n) = \psi_n(y^n)$
- What is the condition on $p_{S,T}$ and W so that

$$\lim_{n\to\infty} \mathbb{P}[(\hat{S}^n, \hat{T}^n) \neq (S^n, T^n)] = 0$$

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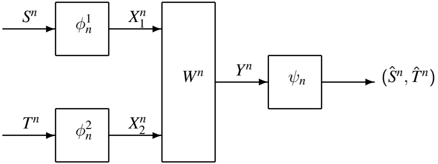

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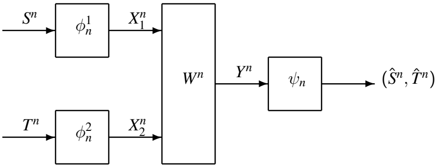

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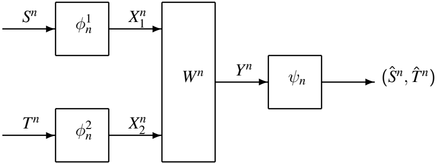

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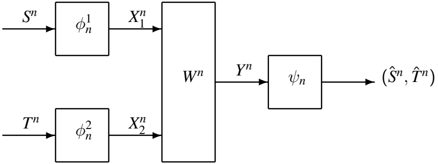

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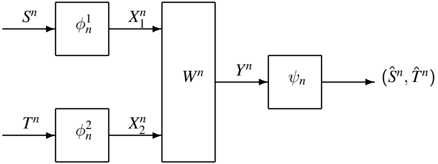

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- Unlike the MAC for channel coding, S^n and T^n can be correlated
- The ratio of channel uses to source symbols is assumed to be 1
- Second-order coding analysis has been done for
  - Slepian-Wolf (Tan-Kosut 2012) – Will review
  - DM-MAC (Tan-Kosut 2012, Huang-Moulin 2012 and MolavianJazi-Laneman 2012) – Will review
  - JSCC (Wang-Ingber-Kochman 2011, Kostina-Verdú 2012)

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- Our result states that (S,T) is (n,ϵ)-transmissible over W if

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- Surprisingly, at blocklength n, two inequalities that can be removed for CES cannot be eliminated in our second-order characterization

A Brief Recap I

- In the asymptotic setting, by combining Slepian-Wolf coding and by encoding the compressed bits using a multiple-access channel code, if there exists $p_Q(q), p_{X_1|Q}(x_1|q), p_{X_2|Q}(x_2|q)$ such that

$$H(S|T) < I(X_1; Y|X_2, Q)$$
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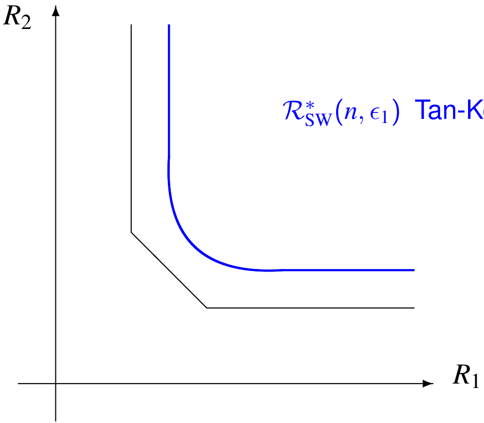
- CES showed that this is strictly suboptimal

Slepian-Wolf and MAC finite blocklength regions



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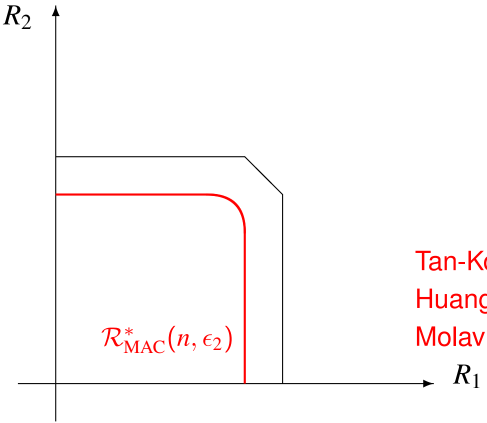

Tan-Kosut (2012)

Huang-Moulin (2012)

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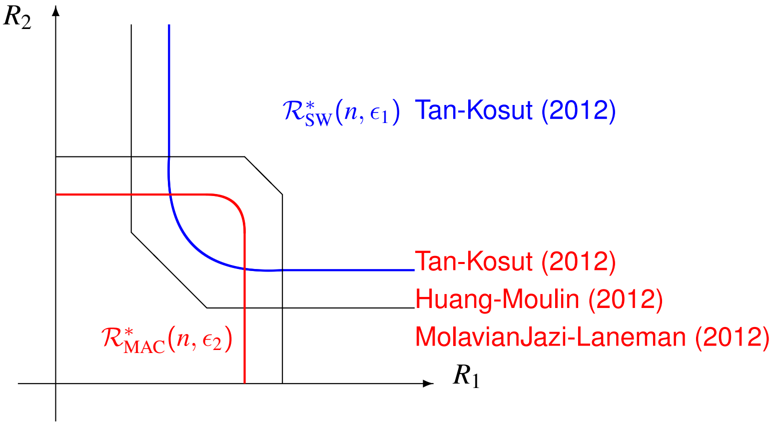

If $\mathcal{R}^*_{\mathrm{SW}}(n, \epsilon_1) \cap \mathcal{R}^*_{\mathrm{MAC}}(n, \epsilon_2) \neq \emptyset$ for some $p(q), p(x_1|q), p(x_2|q)$, then source is $(n, \tilde{\epsilon})$-transmissible using a separation strategy where

$$\tilde{\epsilon} = \epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2$$

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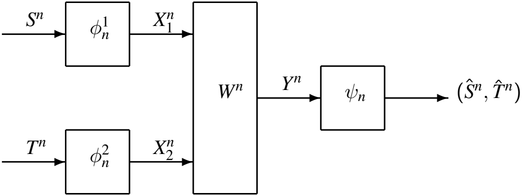
- Generalization of the CES region when we only allow a blocklength n?

Formal Definition of a Code



Definition

An $(|\mathcal{S}|^n, |\mathcal{T}|^n, n, \epsilon)$ joint source-channel code for transmitting the correlated source (S, T) over the MAC W consists of two encoders $\phi_n^1 : \mathcal{S}^n \to \mathcal{X}_1^n$, $\phi_n^2 : \mathcal{T}^n \to \mathcal{X}_2^n$ and a decoder $\psi_n : \mathcal{Y}^n \to \mathcal{S}^n \times \mathcal{T}^n$ s.t.

$$\mathbb{P}[(\hat{S}^n, \hat{T}^n) \neq (S^n, T^n)] \leq \epsilon$$

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Define the set

$$\mathcal{S}_{\mathbf{V}}(\epsilon) = \left\{\mathbf{z} \in \mathbb{R}^d : \mathbb{P}(\mathbf{Z} \le \mathbf{z}) \ge 1 - \epsilon\right\}$$

where $\mathbf{Z} \sim \mathcal{N}(\mathbf{0}, \mathbf{V})$.

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- Analogue of the Q^{-1} function
- If $d = 1$, $\mathcal{S}_{\sigma^2}(\epsilon) = [\sigma Q^{-1}(\epsilon), \infty)$
- This set was also used to characterize the (n, ϵ)-rate region for Slepian-Wolf coding

What's this special set $\mathcal{S}_{\mathbf{V}}(\epsilon)$?

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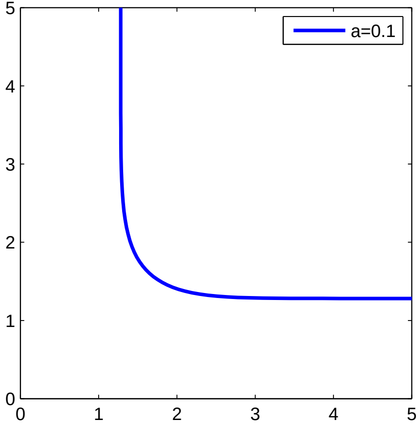

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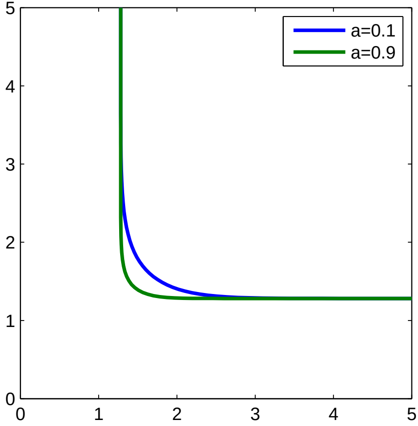

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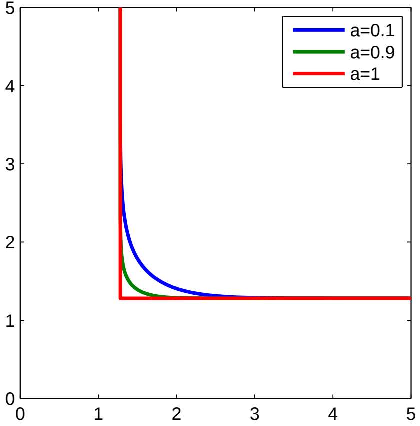

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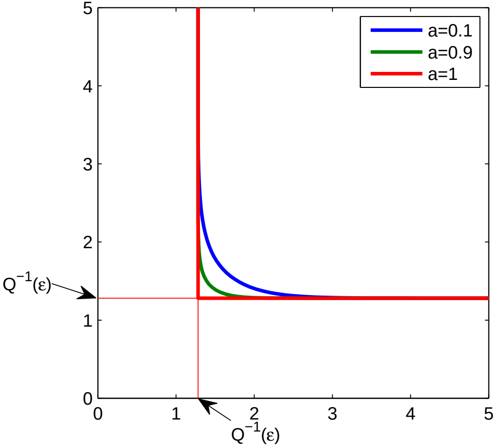

Main Result

Theorem

Let K be the common part of (S, T). If for sufficiently large n, there exists distributions $p_C(c), p_{X_1|S,C}(x_1|s, c)$ and $p_{X_2|T,C}(x_2|t, c)$ such that

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where the information dispersion matrix $\mathbf{V}(C, X_1, X_2)$ is the covariance of the random vector of differences between information and entropy densities (see paper).

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Can be improved using a time-sharing rv Q [Huang and Moulin, 2012]

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- All constraints are coupled
- Gaussian approximations and Chernoff bounds in the proof
- But two extra inequalities that cannot be removed when n is finite

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$$I(X_2, C; Y|X_3, S) < H(T|S) + \sqrt{\frac{V_4}{n}}$$

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- Depends on dispersions V_1 and V_3 and their correlations

Comments on Main Result II

- Recall in the classical case the first constraint is

$$H(S|T) < I(X_1; Y|X_2, T, C)$$

which immediately implies "third constraint"

$$H(S|T) < I(X_1, C; Y|X_2, T)$$

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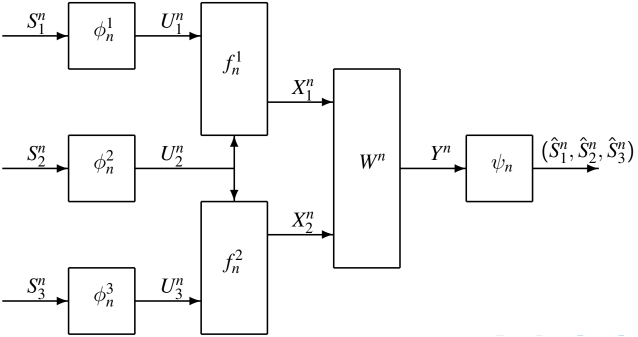
- Depends on dispersions V_1 and V_3 and their correlations
- But region reduces to CES when $n \to \infty$

- Consider transmitting a 3-DMS (S_1, S_2, S_3) over a MAC $W : \mathcal{X}_1 \times \mathcal{X}_2 \to \mathcal{Y}$



Proof Idea II: An Auxiliary System (Ahlswede-Han)

- Input 1 $X_1^n = f_n^1(U_1^n, U_2^n)$ and U_j^n is a stoc. function of S_j^n
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- Results in seven inequalities of which one can be eliminated
- Because we don't have to explicitly recover the common part $S_2 = K$

Proof Idea III: (n, ϵ)-Transmissibility for Ahlswede-Han

Theorem ((n, ϵ)-Transmissibility for the joint source (S_1, S_2, S_3))

If for sufficiently large n, there exists distributions $p_{U_1|S_1}, p_{U_2|S_2}, p_{U_3|S_3}$ and functions f_1 and f_2 such that

$$[I(U_{\mathcal{A}}; Y|U_{\mathcal{A}^c}, S_{\mathcal{A}^c}) - H(S_{\mathcal{A}}|S_{\mathcal{A}^c}) : \emptyset \neq \mathcal{A} \subset [3]] \in \frac{\mathcal{S}_{\mathbf{V}(U_1,U_2,U_3)}(\epsilon)}{\sqrt{n}} + \frac{\log n}{n}\mathbf{1}_7,$$

where the information dispersion matrix $\mathbf{V}(U_1, U_2, U_3)$ *is the covariance of the difference between the information and entropy densities*

$$\left[\log \frac{p_{Y|U_{\mathcal{A}},U_{\mathcal{A}^c},S_{\mathcal{A}^c}}(Y|U_{\mathcal{A}}, U_{\mathcal{A}^c}, S_{\mathcal{A}^c})}{p_{Y|U_{\mathcal{A}^c},S_{\mathcal{A}^c}}(Y|U_{\mathcal{A}^c}, S_{\mathcal{A}^c})} - \log \frac{1}{p_{S_{\mathcal{A}}|S_{\mathcal{A}^c}}(S_{\mathcal{A}}|S_{\mathcal{A}^c})} : \emptyset \neq \mathcal{A} \subset [3] \right],$$

then the source (S_1, S_2, S_3) is (n, ϵ)-transmissible over W.

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where $\mathcal{T}_{\delta_n}^{(n)}$ is the intersection of all typical sets

$$\mathcal{T}_{\delta_n}^{(n)}(\mathcal{A}) := \Bigg\{ (s_1^n, s_2^n, s_3^n, u_1^n, u_2^n, u_3^n, y^n) : \frac{1}{n}\log\frac{p_{Y^n|U_{\mathcal{A}}^n, U_{\mathcal{A}^c}^n, S_{\mathcal{A}^c}^n}(y^n|u_{\mathcal{A}}^n(s_{\mathcal{A}}^n), u_{\mathcal{A}^c}^n(s_{\mathcal{A}^c}^n), s_{\mathcal{A}^c}^n)}{p_{Y^n|U_{\mathcal{A}}^n, U_{\mathcal{A}^c}^n}(y^n|u_{\mathcal{A}}^n(s_{\mathcal{A}}^n), u_{\mathcal{A}^c}^n(s_{\mathcal{A}^c}^n))} - \frac{1}{n}\log\frac{1}{p_{S_{\mathcal{A}}^n|S_{\mathcal{A}^c}^n}(s_{\mathcal{A}}^n|s_{\mathcal{A}^c}^n)} \geq \delta_n \Bigg\}$$

Proof Idea V: (n, ϵ)-Transmissibility for Ahlswede-Han

- The error events are

$$\mathcal{E}_0 := \{(S_1^n, S_2^n, S_3^n, U_1^n(S_1^n), U_2^n(S_2^n), U_3^n(S_3^n), Y^n) \notin \mathcal{T}_{\delta_n}^{(n)}\}$$

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- By multi-dimensional Berry-Essèen theorem and the choice of source (S_1, S_2, S_3) and channel W

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- By Chernoff bounds, for every $\emptyset \neq \mathcal{A} \subset [3]$,

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- By using alternative techniques, we could potentially relax our condition on (n, ϵ)-transmissibility