Exact Error and Erasure Exponents for the Asymmetric Broadcast Channel

Daming Cao [1] Vincent Y. F. Tan [2]

[1]Southeast University

[2]National University of Singapore

June 2018

Decoding with Erasure in the Point-to-Point Channel

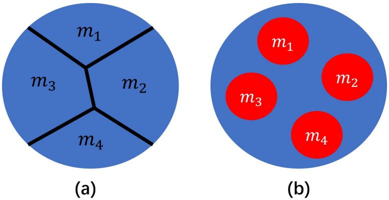


Figure: Representation of typical decision regions:(a) ordinary decoding, (b) erasure option

Decoding with Erasure in the Point-to-Point Channel

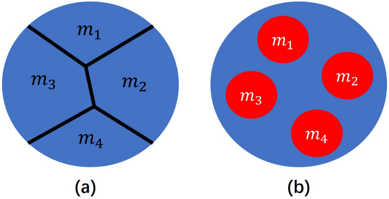


Figure: Representation of typical decision regions:(a) ordinary decoding, (b) erasure option

- Total error and undetected error.

$$\Pr\{\mathcal{E}^{\mathrm{t}}\} = \frac{1}{M}\sum_{m}\sum_{y^n \notin D(m)} \Pr\{y^n|x^n(m)\}$$

$$\Pr\{\mathcal{E}^{\mathrm{u}}\} = \frac{1}{M}\sum_{m}\sum_{m' \neq m}\sum_{y^n \in D(m')} \Pr\{y^n|x^n(m)\}$$

Decoding with Erasure in the Point-to-Point Channel

- Using the Neyman-Pearson theorem, Forney[1] showed that the optimal trade-off between the average total and undetected error probabilities is attained by the decoding regions

$$y^n \in \mathcal{D}(m) \quad \Longleftrightarrow \quad \ln \frac{\Pr\{y^n|x^n(m)\}}{\sum_{m' \neq m} \Pr\{y^n|x^n(m')\}} \geq nT$$

[1]G. Forney, "Exponential error bounds for erasure, list, and decision feedback schemes," IEEE Trans. on Inform. Th., vol. 14, no. 2, pp. 206–220, Mar 1968.

[2]A. Somekh-Baruch and N. Merhav, "Exact random coding exponents for erasure decoding," IEEE Trans. on Inform. Th., vol. 57, no. 10, pp. 6444–6454, Oct 2011.

Decoding with Erasure in the Point-to-Point Channel

- Using the Neyman-Pearson theorem, Forney[1] showed that the optimal trade-off between the average total and undetected error probabilities is attained by the decoding regions

$$y^n \in \mathcal{D}(m) \quad \Longleftrightarrow \quad \ln \frac{\Pr\{y^n|x^n(m)\}}{\sum_{m'\neq m} \Pr\{y^n|x^n(m')\}} \geq nT$$

- Somekh-Baruch and Merhav[2] derived the *exact* random coding exponents for erasure decoding in the single-user DMC.

[1] G. Forney, "Exponential error bounds for erasure, list, and decision feedback schemes," IEEE Trans. on Inform. Th., vol. 14, no. 2, pp. 206–220, Mar 1968.

[2] A. Somekh-Baruch and N. Merhav, "Exact random coding exponents for erasure decoding," IEEE Trans. on Inform. Th., vol. 57, no. 10, pp. 6444–6454, Oct 2011.

Decoding with Erasure in the Asymmetric Broadcast Channel

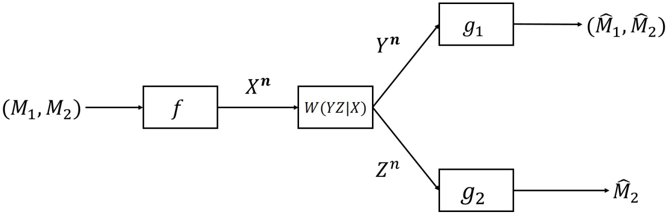


Figure: Asymmetric Broadcast Channel

Decoding with Erasure in the Asymmetric Broadcast Channel

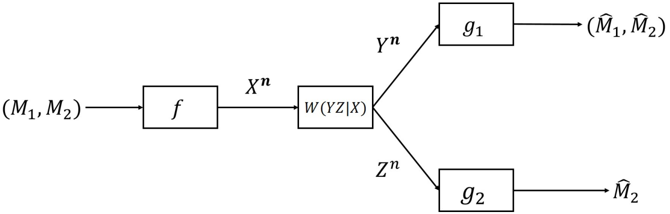


Figure: Asymmetric Broadcast Channel

- Decoding with erasure option.

$$g_1 : \mathcal{Y}^n \to (\mathcal{M}_1 \cup \{\mathrm{e}_1\}) \times (\mathcal{M}_2 \cup \{\mathrm{e}_2\})$$
$$g_2 : \mathcal{Z}^n \to \mathcal{M}_2 \cup \{\mathrm{e}_2\}$$

The total error and undetected error

Define the average total and undetected error probabilities at terminal $\mathcal{Y}$ as follows:

$$e_Y^{\mathrm{t}} \triangleq \frac{1}{M_1 M_2} \sum_{(m_1,m_2)\in\mathcal{M}_1\times\mathcal{M}_2} W_{\mathcal{Y}}^n \left(\mathcal{D}_{m_1,m_2}^c \,\middle|\, x^n(m_1,m_2)\right)$$

$$e_Y^{\mathrm{u}} \triangleq \frac{1}{M_1 M_2} \sum_{(m_1,m_2)\in\mathcal{M}_1\times\mathcal{M}_2} W_{\mathcal{Y}}^n \Bigg(\bigcup_{(\hat{m}_1,\hat{m}_2)\neq(m_1,m_2)} \mathcal{D}_{\hat{m}_1\hat{m}_2} \,\Bigg|\, x^n(m_1,m_2) \Bigg)$$

The total error and undetected error

Define the average total and undetected error probabilities at terminal $\mathcal{Y}$ as follows:

$$e_Y^{\mathrm{t}} \triangleq \frac{1}{M_1 M_2} \sum_{(m_1,m_2)\in\mathcal{M}_1\times\mathcal{M}_2} W_{\mathcal{Y}}^n\left(\mathcal{D}_{m_1,m_2}^c \,\middle|\, x^n(m_1,m_2)\right)$$

$$e_Y^{\mathrm{u}} \triangleq \frac{1}{M_1 M_2} \sum_{(m_1,m_2)\in\mathcal{M}_1\times\mathcal{M}_2} W_{\mathcal{Y}}^n\left(\bigcup_{(\hat{m}_1,\hat{m}_2)\neq(m_1,m_2)} \mathcal{D}_{\hat{m}_1\hat{m}_2} \,\middle|\, x^n(m_1,m_2)\right)$$

Moreover,

$$e_j^{\mathrm{t}} \triangleq \frac{1}{M_1 M_2} \sum_{(m_1,m_2)\in\mathcal{M}_1\times\mathcal{M}_2} W_{\mathcal{Y}}^n\left(\mathcal{D}_{m_j}^c \,\middle|\, x^n(m_1,m_2)\right)$$

$$e_j^{\mathrm{u}} \triangleq \frac{1}{M_1 M_2} \sum_{(m_1,m_2)\in\mathcal{M}_1\times\mathcal{M}_2} W_{\mathcal{Y}}^n\left(\bigcup_{\hat{m}_j\in\mathcal{M}_j\setminus\{m_j\}} \mathcal{D}_{\hat{m}_j} \,\middle|\, x^n(m_1,m_2)\right).$$

where $j = 1, 2$.

The optimal decoding regions

- $\mathcal{D}^*_{m_1 m_2} \triangleq \left\{ y^n : \ln \frac{W^n_{\mathcal{Y}}(y^n | x^n(m_1, m_2))}{\sum_{(m_1', m_2') \neq (m_1, m_2)} W^n_{\mathcal{Y}}(y^n | x^n(m_1', m_2'))} \geq nT \right\}$

The optimal decoding regions

- $\mathcal{D}^*_{m_1 m_2} \triangleq \left\{ y^n : \ln \frac{W^n_{\mathcal{Y}}(y^n|x^n(m_1,m_2))}{\sum_{(m_1',m_2')\neq(m_1,m_2)} W^n_{\mathcal{Y}}(y^n|x^n(m_1',m_2'))} \geq nT \right\}$

- The average channel of a sub-codebook $\mathcal{C}_1(m_1) = \{x^n(m_1, m_2) : m_2 \in \mathcal{M}_2\}$:

$$\Pr(y^n|\mathcal{C}_1(m_1)) \triangleq \frac{1}{M_2} \sum_{m_2 \in \mathcal{M}_2} W^n_{\mathcal{Y}}(y^n|x^n(m_1, m_2))$$

The optimal decoding regions

- $\mathcal{D}^*_{m_1 m_2} \triangleq \left\{ y^n : \ln \frac{W^n_{\mathcal{Y}}(y^n|x^n(m_1,m_2))}{\sum_{(m_1',m_2')\neq(m_1,m_2)} W^n_{\mathcal{Y}}(y^n|x^n(m_1',m_2'))} \geq nT \right\}$

- The average channel of a sub-codebook $\mathcal{C}_1(m_1) = \{x^n(m_1, m_2) : m_2 \in \mathcal{M}_2\}$:

$$\Pr(y^n|\mathcal{C}_1(m_1)) \triangleq \frac{1}{M_2} \sum_{m_2 \in \mathcal{M}_2} W^n_{\mathcal{Y}}(y^n|x^n(m_1, m_2))$$

- $\mathcal{D}^*_{m_1} \triangleq \left\{ y^n : \ln \frac{\Pr(y^n|\mathcal{C}_1(m_1))}{\sum_{m_1' \neq m_1} \Pr(y^n|\mathcal{C}_j(m_1'))} \geq nT \right\}$

The optimal decoding regions

- $\mathcal{D}^*_{m_1 m_2} \triangleq \left\{ y^n : \ln \frac{W^n_{\mathcal{Y}}(y^n|x^n(m_1,m_2))}{\sum_{(m_1',m_2')\neq(m_1,m_2)} W^n_{\mathcal{Y}}(y^n|x^n(m_1',m_2'))} \geq nT \right\}$

- The average channel of a sub-codebook $\mathcal{C}_1(m_1) = \{x^n(m_1, m_2) : m_2 \in \mathcal{M}_2\}$:

$$\Pr(y^n|\mathcal{C}_1(m_1)) \triangleq \frac{1}{M_2} \sum_{m_2 \in \mathcal{M}_2} W^n_{\mathcal{Y}}(y^n|x^n(m_1, m_2))$$

- $\mathcal{D}^*_{m_1} \triangleq \left\{ y^n : \ln \frac{\Pr(y^n|\mathcal{C}_1(m_1))}{\sum_{m_1' \neq m_1} \Pr(y^n|\mathcal{C}_j(m_1'))} \geq nT \right\}$

- If T increases, the decoding regions become small. Moreover, the total error probabilities increase and the undetected error probabilities decrease.

Problem Formulation-Encoder

- Capacity region of the Asymmetric Broadcast Channel is well known [1] and is achieved with superposition coding.

[1] J. Korner and K. Marton, "General broadcast channels with degraded message sets," IEEE Trans. on Inform. Th., vol. 23, no. 1, pp. 60–64, Jan 1977.

Problem Formulation-Encoder

- Capacity region of the Asymmetric Broadcast Channel is well known [1] and is achieved with superposition coding.

Random coding with superposition structure

- Randomly generate $M_2 = e^{nR_2}$ **"cloud centers"** $\{U^n(m_2) : m_2 \in \mathcal{M}_2 = [M_2]\}$ according to the distribution

$$P(u^n) \triangleq \prod_{i=1}^{n} P_U(u_i).$$

[1] J. Korner and K. Marton, "General broadcast channels with degraded message sets," IEEE Trans. on Inform. Th., vol. 23, no. 1, pp. 60–64, Jan 1977.

Problem Formulation-Encoder

- Capacity region of the Asymmetric Broadcast Channel is well known [1] and is achieved with superposition coding.

Random coding with superposition structure

- Randomly generate $M_2 = e^{nR_2}$ **"cloud centers"** $\{U^n(m_2) : m_2 \in \mathcal{M}_2 = [M_2]\}$ according to the distribution

$$P(u^n) \triangleq \prod_{i=1}^{n} P_U(u_i).$$

- For each cloud center $U^n(m_2)$, randomly generate $M_1 = e^{nR_1}$ **"satellite"** codewords $\{X^n(m_1, m_2) : m_1 \in \mathcal{M}_1 = [M_1]\}$ according to the conditional distribution

$$P(x^n|u^n) := \prod_{i=1}^{n} P_{X|U}(x_i|u_i)$$

[1] J. Korner and K. Marton, "General broadcast channels with degraded message sets," IEEE Trans. on Inform. Th., vol. 23, no. 1, pp. 60–64, Jan 1977.

The Exact Error and Erasure Exponents

We would like to find the *exact* error exponents E_j^{t}, E_j^{u}, E_Y^{t} and E_Y^{u}, $j = 1, 2$ as follow

$$E_1^{\mathrm{t}}(R_1, R_2, T) \triangleq \limsup_{n\to\infty} \left[-\frac{1}{n} \ln \mathbb{E}_{\mathcal{C}}[e_1^{\mathrm{t}}] \right],$$

where the expectation is taken with respect to the randomness of the codebook $\mathcal{C}$, and similarly for the other exponents $E_1^{\mathrm{u}}, E_Y^{\mathrm{t}}, E_Y^{\mathrm{t}}, E_2^{\mathrm{t}}$, and E_2^{u}.

Main Results-Theorem 1

The error exponents E_1^{t}, E_1^{u}, E_Y^{t} and E_Y^{u} are given by

$$E_1^{\mathrm{t}} = E_Y^{\mathrm{t}} = \min\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{b}}\}, \quad \text{and} \quad E_1^{\mathrm{u}} = E_Y^{\mathrm{u}} = E_1^{\mathrm{t}} + T$$

Main Results-Theorem 1

The error exponents E_1^{t}, E_1^{u}, E_Y^{t} and E_Y^{u} are given by

$$E_1^{\mathrm{t}} = E_Y^{\mathrm{t}} = \min\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{b}}\}, \quad \text{and} \quad E_1^{\mathrm{u}} = E_Y^{\mathrm{u}} = E_1^{\mathrm{t}} + T$$

where

$$\Psi_{\mathrm{a}} \triangleq \min_{\hat{Q}_{UXY}} \left[D(\hat{Q}_{UXY} \| P_{UXY}) + \min_{Q_{UX|Y} \in \mathcal{L}_1(\hat{Q}_{UXY}, R_1, R_2, T)} \Phi(Q_{UX|Y}\hat{Q}_Y, R_1, R_2) \right]$$

$$\Psi_{\mathrm{b}} \triangleq \min_{\hat{Q}_{UXY}} \left[D(\hat{Q}_{UXY} \| P_{UXY}) + \min_{Q_{X|UY} \in \mathcal{L}_2(\hat{Q}_{UXY}, R_1, T)} |\beta(Q_{X|UY}\hat{Q}_{UY}, R_1)|_+ \right]$$

and the sets $\mathcal{L}_1$ and $\mathcal{L}_2$ are defined as

$$\mathcal{L}_1(\hat{Q}_{UXY}, R_1, R_2, T) \triangleq \left\{ Q_{UX|Y} : \mathbb{E}_Q \ln \frac{1}{W_{\mathcal{Y}}} + \mathbb{E}_{\hat{Q}} \ln W_{\mathcal{Y}} - T \leq \Delta(Q, R_1, R_2) \right\}$$

$$\mathcal{L}_2(\hat{Q}_{UXY}, R_1, T) \triangleq \left\{ Q_{X|UY} : \mathbb{E}_Q \ln \frac{1}{W_{\mathcal{Y}}} + \mathbb{E}_{\hat{Q}} \ln W_{\mathcal{Y}} - T \leq \big| -\beta(Q, R_1) \big|_+ \right\}$$

where Q in $\mathcal{L}_1$ is equal to $Q = Q_{UX|Y}\hat{Q}_Y$ and Q in $\mathcal{L}_2$ is equal to $Q = Q_{X|UY}\hat{Q}_{UY}$,

Remarks for Theorem 1

- The meaning of $E_1^{\mathrm{t}} = E_Y^{\mathrm{t}}$.

Remarks for Theorem 1

- The meaning of $E_1^{\mathrm{t}} = E_Y^{\mathrm{t}}$.
- Construct a sub-optimal decoding region $\tilde{\mathcal{D}}(m_1)$ from the joint decoding region $\mathcal{D}^*(m_1, m_2)$

$$\tilde{\mathcal{D}}(m_1) = \bigcup_{m_2} \mathcal{D}^*(m_1, m_2)$$

Remarks for Theorem 1

- The meaning of $E_1^{\mathrm{t}} = E_Y^{\mathrm{t}}$.
- Construct a sub-optimal decoding region $\tilde{\mathcal{D}}(m_1)$ from the joint decoding region $\mathcal{D}^*(m_1, m_2)$

$$\tilde{\mathcal{D}}(m_1) = \bigcup_{m_2} \mathcal{D}^*(m_1, m_2)$$

- The event that the user $\mathcal{Y}$ decodes the correct private message m_1 but the wrong common message m_2.

Remarks for Theorem 1

- The meaning of $E_1^{\mathrm{t}} = E_Y^{\mathrm{t}}$.
- Construct a sub-optimal decoding region $\tilde{\mathcal{D}}(m_1)$ from the joint decoding region $\mathcal{D}^*(m_1, m_2)$
$$\tilde{\mathcal{D}}(m_1) = \bigcup_{m_2} \mathcal{D}^*(m_1, m_2)$$
- The event that the user $\mathcal{Y}$ decodes the correct private message m_1 but the wrong common message m_2.
- There is no loss in optimality of using this joint decoding region $\mathcal{D}^*(m_1, m_2)$ for decoding only message m_1

Remarks for Theorem 1

The closed form expression of Ψ_{b}

$$\Psi_{\mathrm{b}} \triangleq \min_{\hat{Q}_{UXY}} \left[D(\hat{Q}_{UXY} \| P_{UXY}) + \min_{Q_{X|UY} \in \mathcal{L}_2(\hat{Q}_{UXY}, R_1, T)} |\beta(Q_{X|UY}\hat{Q}_{UY}, R_1)|_+ \right]$$

$$\beta(Q, R_1) \triangleq D(Q_{X|U} \| P_{X|U} | Q_U) + I_Q(X; Y|U) - R_1$$

Remarks for Theorem 1

The closed form expression of Ψ_{b}

$$\Psi_{\mathrm{b}} \triangleq \min_{\hat{Q}_{UXY}} \left[D(\hat{Q}_{UXY} \| P_{UXY}) + \min_{Q_{X|UY} \in \mathcal{L}_2(\hat{Q}_{UXY}, R_1, T)} |\beta(Q_{X|UY}\hat{Q}_{UY}, R_1)|_+ \right]$$

$$\beta(Q, R_1) \triangleq D(Q_{X|U} \| P_{X|U} | Q_U) + I_Q(X; Y|U) - R_1$$

- Ψ_{b} does not depend on R_2, and its expression is averaged over the random variable U.

Remarks for Theorem 1

The closed form expression of $\Psi_{\rm b}$

$$\Psi_{\rm b} \triangleq \min_{\hat{Q}_{UXY}} \left[D(\hat{Q}_{UXY} \| P_{UXY}) + \min_{Q_{X|UY} \in \mathcal{L}_2(\hat{Q}_{UXY}, R_1, T)} |\beta(Q_{X|UY} \hat{Q}_{UY}, R_1)|_+ \right]$$

$$\beta(Q, R_1) \triangleq D(Q_{X|U} \| P_{X|U} | Q_U) + I_Q(X; Y|U) - R_1$$

- $\Psi_{\rm b}$ does not depend on R_2, and its expression is averaged over the random variable U.
- If a genie gives the true common message m_2 to the user $\mathcal{Y}$, all codewords are conditioned on a particular $u^n(m_2)$ and are generated according to a conditional distribution $P_{X|U}$.

Remarks for Theorem 1

The closed form expression of $\Psi_{\rm b}$

$$\Psi_{\rm b} \triangleq \min_{\hat{Q}_{UXY}} \left[D(\hat{Q}_{UXY} \| P_{UXY}) + \min_{Q_{X|UY} \in \mathcal{L}_2(\hat{Q}_{UXY}, R_1, T)} |\beta(Q_{X|UY}\hat{Q}_{UY}, R_1)|_+ \right]$$

$$\beta(Q, R_1) \triangleq D(Q_{X|U} \| P_{X|U} | Q_U) + I_Q(X; Y|U) - R_1$$

- $\Psi_{\rm b}$ does not depend on R_2, and its expression is averaged over the random variable U.
- If a genie gives the true common message m_2 to the user $\mathcal{Y}$, all codewords are conditioned on a particular $u^n(m_2)$ and are generated according to a conditional distribution $P_{X|U}$.
- Compare to the single-user DMC case, when $T = 0$, $\Psi_{\rm b}$ can be viewed as the "*conditional*" random coding error exponent.

Main Results-Theorem 2

The error exponents E_2^{t} and E_2^{u} are given by

$$E_2^{\mathrm{t}} = \max\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{c}}\}, \quad \text{and} \quad E_2^{\mathrm{u}} = E_2^{\mathrm{t}} + T,$$

Main Results-Theorem 2

The error exponents E_2^{t} and E_2^{u} are given by

$$E_2^{\mathrm{t}} = \max\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{c}}\}, \quad \text{and} \quad E_2^{\mathrm{u}} = E_2^{\mathrm{t}} + T,$$

where

$$\Psi_{\mathrm{c}} \triangleq \min_{\hat{Q}_{UXY}} \Big[D(\hat{Q}_{UXY} \| P_{UXY}) + \min_{Q_{UX|Y} \in \mathcal{L}_3(\hat{Q}_{UXY}, R_1, R_2, T)} \Phi(Q_{UX|Y}\hat{Q}_Y, R_1, R_2) \Big]$$

with

$$\mathcal{L}_3(\hat{Q}_{UXY}, R_1, R_2, T) \triangleq \left\{ Q_{UX|Y} : \mathbb{E}_Q \ln \frac{1}{W_{\mathcal{Y}}} + s_0(\hat{Q}_{UXY}, R_1) - T \leq \Delta(Q, R_1, R_2) \right\}$$

and where Q in $\mathcal{L}_3$ is equal to $Q = Q_{UX|Y}\hat{Q}_Y$, and

$$s_0(\hat{Q}_{UXY}, R_1) \triangleq - \min_{\tilde{Q}_{X|UY} : \beta(\tilde{Q}, R_1) \leq 0} \left[\beta(\tilde{Q}, R_1) - \mathbb{E}_{\tilde{Q}} \ln W_{\mathcal{Y}} \right]$$

and where $\tilde{Q}$ in s_0 is equal to $\tilde{Q} = \tilde{Q}_{X|UY}\hat{Q}_{UY}$.

Remarks for Theorem 2

- The intuition of the maximum in $E_2^{\mathrm{t}} = \max\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{c}}\}$

Remarks for Theorem 2

- The intuition of the maximum in $E_2^{\mathrm{t}} = \max\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{c}}\}$
- The user $\mathcal{Y}$ has two strategies to decode the message $\hat{m}_2$.

Remarks for Theorem 2

- The intuition of the maximum in $E_2^{\mathrm{t}} = \max\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{c}}\}$
- The user $\mathcal{Y}$ has two strategies to decode the message $\hat{m}_2$.
  1. The user $\mathcal{Y}$ decodes the pair of messages $(\hat{m}_1, \hat{m}_2)$ and declares the message $\hat{m}_2$. And this strategy corresponds to the exponent Ψ_{a}.

Remarks for Theorem 2

- The intuition of the maximum in $E_2^{\mathrm{t}} = \max\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{c}}\}$
- The user $\mathcal{Y}$ has two strategies to decode the message $\hat{m}_2$.
  1. The user $\mathcal{Y}$ decodes the pair of messages $(\hat{m}_1, \hat{m}_2)$ and declares the message $\hat{m}_2$. And this strategy corresponds to the exponent Ψ_{a}.
  2. The user $\mathcal{Y}$ decodes the entire sub-codebook for the common message m_2, i.e., $\mathcal{C}_2(m_2) = \{X^n(m_1, m_2) : m_1 \in [M_1]\}$. And this strategy corresponds to the exponent Ψ_{c}.

Remarks for Theorem 2

- The intuition of the maximum in $E_2^{\mathrm{t}} = \max\{\Psi_{\mathrm{a}}, \Psi_{\mathrm{c}}\}$
- The user $\mathcal{Y}$ has two strategies to decode the message $\hat{m}_2$.
  1. The user $\mathcal{Y}$ decodes the pair of messages $(\hat{m}_1, \hat{m}_2)$ and declares the message $\hat{m}_2$. And this strategy corresponds to the exponent Ψ_{a}.
  2. The user $\mathcal{Y}$ decodes the entire sub-codebook for the common message m_2, i.e., $\mathcal{C}_2(m_2) = \{X^n(m_1, m_2) : m_1 \in [M_1]\}$. And this strategy corresponds to the exponent Ψ_{c}.
- Loosely speaking, when R_1 is large and the exponent Ψ_{c} achieves the maximum in E_2^{t}, the user $\mathcal{Y}$ is more likely to decode the "cloud center" $U^n(m_2)$ according to the "test channel"

$$W_{Y|U}(y|u) \triangleq \sum_x W_{\mathcal{Y}}(y|x) P_{X|U}(x|u)$$

rather than the average channel

$$\Pr(y^n|\mathcal{C}_2(m_2)) \triangleq \frac{1}{M_1} \sum_{m_1 \in \mathcal{M}_1} W_{\mathcal{Y}}^n(y^n|x^n(m_1, m_2))$$

Numerical Evaluations

- In the paper, we show that the minimizations required to evaluate these error exponents can be cast as convex optimization problems, and thus, can be solved efficiently.

Numerical Evaluations

- In the paper, we show that the minimizations required to evaluate these error exponents can be cast as convex optimization problems, and thus, can be solved efficiently.

- We consider binary symmetric channels (BSCs) as follow

$$Y = X \oplus Z_2 \qquad X = U \oplus Z_1$$
$$U \sim \text{Bern}(0.5) \quad Z_1 \sim \text{Bern}(p_1) \quad Z_2 \sim \text{Bern}(p_2)$$

where $U, X, Y, Z_1, Z_2 \in \{0, 1\}$, $p_1 = 0.1$ and $p_2 = 0.2$.

Numerical Evaluations

- In the paper, we show that the minimizations required to evaluate these error exponents can be cast as convex optimization problems, and thus, can be solved efficiently.

- We consider binary symmetric channels (BSCs) as follow

$$Y = X \oplus Z_2 \qquad X = U \oplus Z_1$$
$$U \sim \text{Bern}(0.5) \quad Z_1 \sim \text{Bern}(p_1) \quad Z_2 \sim \text{Bern}(p_2)$$

where $U, X, Y, Z_1, Z_2 \in \{0, 1\}$, $p_1 = 0.1$ and $p_2 = 0.2$.

- The set of achievable rates for M_1 and (M_1, M_2).

$$\{R_1 \leq I(X; Y|U) = 0.07\} \bigcap \{R_1 + R_2 \leq I(X; Y) = 0.19\}.$$

- The set of achievable rates for M_2.

$$\{R_2 \leq I(U; Y) = 0.12\} \bigcup \{R_1 + R_2 \leq I(X; Y) = 0.19\}.$$

Numerical Evaluations

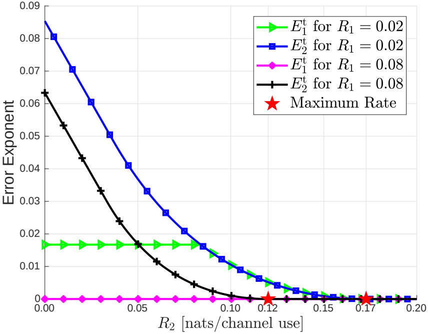
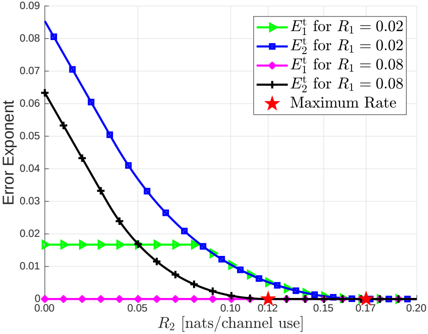


Figure: Total error exponents E_1^{t} and E_2^{t} as a function of R_2 for two different values of R_1 and where the threshold $T = 0$.

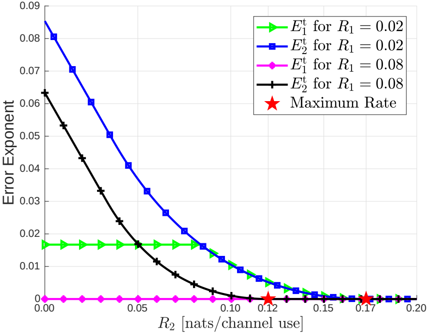
Numerical Evaluations



- The exponents decrease when R_2 increases.

Figure: Total error exponents $E_1^{\rm t}$ and $E_2^{\rm t}$ as a function of R_2 for two different values of R_1 and where the threshold $T = 0$.

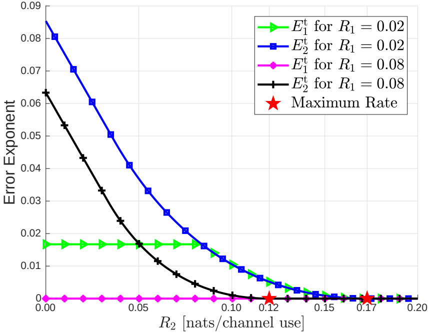
Numerical Evaluations



- The exponents decrease when R_2 increases.
- When R_2 is below a critical value, the green line (E_1^{t}) is horizontal.

Figure: Total error exponents E_1^{t} and E_2^{t} as a function of R_2 for two different values of R_1 and where the threshold $T = 0$.

Numerical Evaluations



- The exponents decrease when R_2 increases.
- When R_2 is below a critical value, the green line (E_1^{t}) is horizontal.
- When R_2 is small, the user $\mathcal{Y}$ can easily decode the true common message m_2.

Figure: Total error exponents E_1^{t} and E_2^{t} as a function of R_2 for two different values of R_1 and where the threshold $T = 0$.

Numerical Evaluations

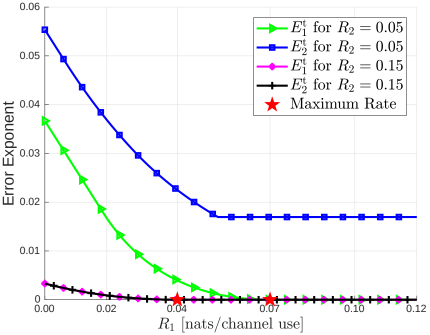
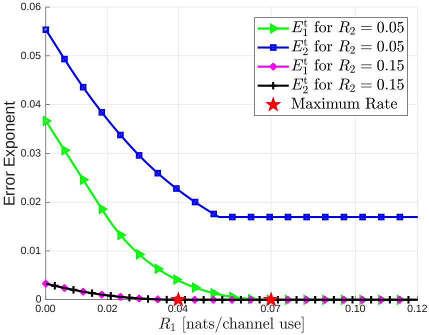


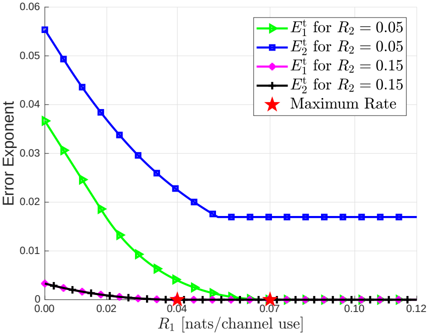
Figure: Total error exponents E_1^{t} and E_2^{t} as a function of R_1 for two different values of R_2 and where the threshold $T = 0$.

Numerical Evaluations



- The exponents decrease when R_2 increases.

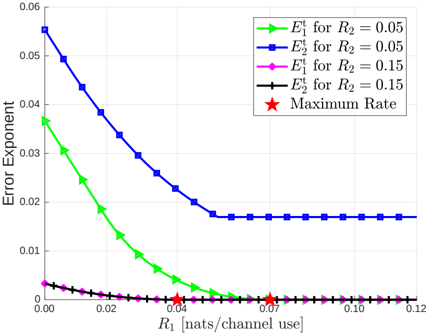
Figure: Total error exponents E_1^{t} and E_2^{t} as a function of R_1 for two different values of R_2 and where the threshold $T = 0$.



- The exponents decrease when R_2 increases.
- When R_1 is above a critical value, the blue line (E_2^t) is horizontal.

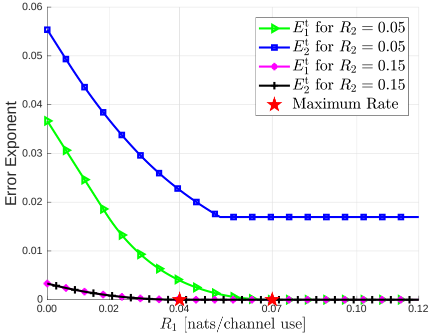
Figure: Total error exponents E_1^t and E_2^t as a function of R_1 for two different values of R_2 and where the threshold $T = 0$.

Numerical Evaluations



- The exponents decrease when R_2 increases.
- When R_1 is above a critical value, the blue line (E_2^{t}) is horizontal.
- When $R_2 \geq I(U;Y)$, the black line (E_2^{t}) decreases to zero.

Figure: Total error exponents E_1^{t} and E_2^{t} as a function of R_1 for two different values of R_2 and where the threshold $T = 0$.



- The exponents decrease when R_2 increases.
- When R_1 is above a critical value, the blue line (E_2^{t}) is horizontal.
- When $R_2 \geq I(U;Y)$, the black line (E_2^{t}) decreases to zero.
- When R_1 is large, the user $\mathcal{Y}$ is more likely to directly decode the "cloud center" $u^n(m_2)$ according to the "test channel" $W_{Y|U}$.

Figure: Total error exponents E_1^{t} and E_2^{t} as a function of R_1 for two different values of R_2 and where the threshold $T = 0$.

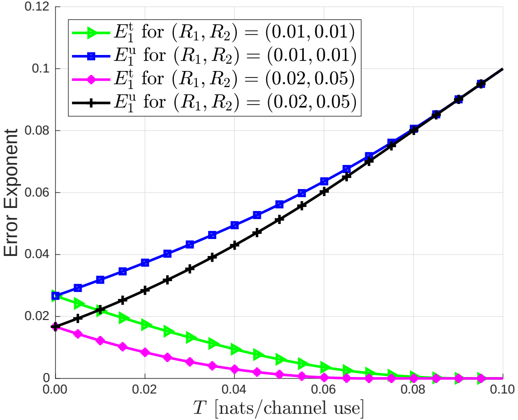


Figure: Total error exponent E_1^{t} and undetected error exponent E_1^{u} for message m_1 as a function of T for two different pairs of (R_1, R_2)

Thank You!

Full version: https://arxiv.org/pdf/1801.05112.pdf