# Asymptotic Coupling and Its Applications in Information Theory

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# Outline

# General Coupling Problem

- A joint distribution $Q_{XY} \in \mathcal{P}(\mathcal{X} \times \mathcal{Y})$ such that $Q_X = P_X, Q_Y = P_Y$ is called a coupling of $P_X, P_Y$.

# General Coupling Problem

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- The set of couplings of $P_X, P_Y$ is defined as

$$C(P_X, P_Y) := \{Q_{XY} \in \mathcal{P}(\mathcal{X} \times \mathcal{Y}) : Q_X = P_X, Q_Y = P_Y\}$$

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- Coupling Problem: Given marginals $P_X$ and $P_Y$ and a real-valued function $g(P_{XY})$, what is the value of

$$\max_{P_{XY} \in C(P_X, P_Y)} g(P_{XY})?$$

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General coupling problem

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| General Coupling Problem | $g(P_{XY})$ |
|---|---|
| Maximal Coupling | $\mathbb{P}(Y = X)$ |
| Maximal Guessing Coupling | $\max_f \mathbb{P}(Y = f(X))$ |
| Minimum Distance Coupling | $-\mathbb{E}[d(X, Y)]$ |
| Minimum Excess-Distortion Coupling | $-\mathbb{P}\{d(X, Y) > D\}$ |

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In our work we consider a large number of random variables, i.e.,

$$X \leftarrow X^n \quad \text{and} \quad Y \leftarrow Y^n \quad \text{as} \quad n \to \infty.$$

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- The Total Variation Distance between $P$ and $Q$ is

$$|P - Q|_{TV} := \frac{1}{2} \sum_x |P(x) - Q(x)|.$$

# Useful Lemma: Maximal Coupling Equality

## Lemma

*Given $P_X$ and $P_Y$, we have*

$$\mathcal{M}(P_X, P_Y) := \max_{P_{XY} \in C(P_X, P_Y)} \mathbb{P}\{Y = X\} = 1 - |P_X - P_Y|_{TV}.$$

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*Furthermore, the (optimal) maximal coupling is*

$$P_{XY}(x,y) = \begin{cases} \min\{P_X(x), P_Y(y)\}, & x = y; \\ q_{x,y}, & x \neq y \end{cases}$$

*where $q_{x,y}$ (for $x \neq y$) can take on any value as long as $P_{XY}$ forms a valid distribution.*

# Useful Lemma: Maximal Coupling Equality

**Proof.**

$$\mathbb{P}\{Y = X\} = \sum_x P_{XY}(x,x) \le \sum_x \min\{P_X(x), P_Y(x)\} = 1 - |P_X - P_Y|_{TV}.$$

Moreover, the "$=$" holds for $P_{XY}$ defined for the maximal coupling equality.

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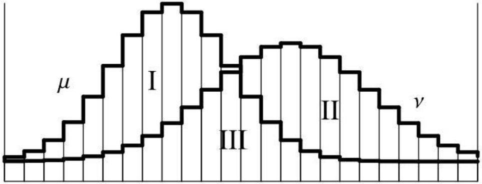


$\mathcal{M}(P_X, P_Y)$ corresponds to Region III$= \sum_x \min\{P_X(x), P_Y(x)\}$

# Maximal Coupling $\mathcal{M}(P_X^n, P_Y^n)$

## Theorem

*If $P_X \neq P_Y$, then given $P_X$ and $P_Y$, we have $\mathcal{M}(P_X^n, P_Y^n) \to 0$ exponentially fast as $n \to \infty$. More explicitly, the exponent is*

$$\lim_{n\to\infty} -\frac{1}{n} \log \mathcal{M}(P_X^n, P_Y^n) = \min_Q \max \{D(Q\|P_X), D(Q\|P_Y)\} .$$

*Note that $Q^*$ is the mid-point of the e-geodesic connecting $P_X$ and $P_Y$.*

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*An optimal product coupling $P_{X^nY^n} = P_{XY}^n$ with $P_{XY}$ achieving $\mathcal{M}(P_X, P_Y)$ only achieves the smaller exponent*

$$-\log \mathcal{M}(P_X, P_Y) = -\log\left(1 - |P_X - P_Y|\right),$$

*which is suboptimal in general.*

# Maximal Guessing Coupling Problem

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$$\mathcal{G}(P_X, P_Y) := \max_{P_{XY} \in C(P_X, P_Y)} \max_f \mathbb{P}\{Y = f(X)\}$$

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- Our question: What is the value of $\lim_{n\to\infty} \mathcal{G}(P_X^n, P_Y^n)$?
- How does the limit depend on $P_X$ and $P_Y$?

# Outline

# Maximal Guessing Coupling Equality

## Lemma

*The maximal guessing coupling problem is equivalent to the distribution approximation problem. That is,*

$$\mathcal{G}(P_X, P_Y) := \max_{P_{XY} \in C(P_X, P_Y)} \max_f \mathbb{P}\{Y = f(X)\}$$
$$= 1 - \min_f |P_Y - P_{f(X)}|_{TV}.$$

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In the problem

$$\min_f |P_Y - P_{f(X)}|_{TV}$$

we try to approximate the distribution of $Y$ by given a random variable $X$ and we design a function $f : \mathcal{X} \to \mathcal{Y}$.

# Maximal Guessing Coupling Equality

## Proof.

$$\mathcal{G}(P_X, P_Y)$$

$$= \max_{P_{XY} \in C(P_X, P_Y)} \max_f \mathbb{P}\{Y = f(X)\} \qquad \text{(Definition)}$$

□

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&= \max_f \max_{P_{XY} \in C(P_X, P_Y)} \mathbb{P}\{Y = f(X)\} && \text{(Exchanging maximizations)}
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# Main Result

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- It was proved
  - If $H(X) > H(Y)$, then $\min_f |P_Y^n - P_{f(X^n)}|_{TV} \to 0$ at least exponentially fast as $n \to \infty$.
  - If $H(X) < H(Y)$, then $\min_f |P_Y^n - P_{f(X^n)}|_{TV} \to 1$ at least exponentially fast as $n \to \infty$.

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- We obtain different exponents for these two convergences by using the method of types.
- We also show that if $H(X) = H(Y)$, then

$$\mathcal{G}(P_X^n, P_Y^n) \geq \mathcal{G}(P_X, P_Y)^n, \qquad \forall n \in \mathbb{N}$$

# Exponents

## Theorem

1. *If* $H(X) > H(Y)$, *then* $\mathcal{G}(P_X^n, P_Y^n) \to 1$ *at least exponentially fast as* $n \to \infty$. *Moreover, the exponent is*

$$\overline{\mathsf{E}}\left(P_X, P_Y\right) := \liminf_{n\to\infty} -\frac{1}{n} \log\left(1 - \mathcal{G}(P_X^n, P_Y^n)\right) \geq \overline{\mathsf{E}}_{\mathsf{iid}}\left(P_X, P_Y\right).$$

*with* $\overline{\mathsf{E}}_{\mathsf{iid}}\left(P_X, P_Y\right) := \dfrac{1}{2} \max_{t\in[0,1]} t\left(H_{1+t}(X) - H_{1-t}\left(Y\right)\right).$

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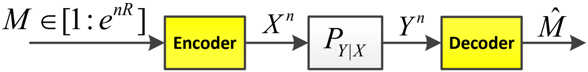
$$\mathsf{E}\,(P_X, P_Y) : = \liminf_{n\to\infty} -\frac{1}{n} \log \mathcal{G}(P_X^n, P_Y^n)$$
$$\geq \sup_{\epsilon\in(0,1)} \min\left\{\frac{1}{3}\epsilon^2 P_X^{(\min)}, \frac{1}{3}\epsilon^2 P_Y^{(\min)}, (1-\epsilon)H(Y) - (1+\epsilon)H(X)\right\}.$$

# Outline

# Applications

- Channel Capacity With Input Distribution Constraint
- Communication with Perfect Stealth/Covert Communications

# Channel Capacity With Input Distribution Constraint




$$X^n \sim P_X^n \qquad \mathbf{P}\left\{M \neq \hat{M}\right\} \to 0$$

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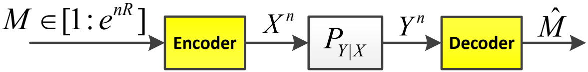


$$X^n \sim P_X^n \qquad \mathbf{P}\left\{M \neq \hat{M}\right\} \to 0$$

- Channel Capacity With Input Distribution Constraint is defined as

$$C(P_X) := \sup\Big\{R : \exists (P_{X^n|M_n}, P_{\widehat{M}_n|Y^n})_{n=1}^{\infty} \text{ s.t.}$$
$$P_{X^n} = P_X^n,$$
$$\lim_{n\to\infty} \mathbb{P}\left\{M_n \neq \widehat{M}_n\right\} = 0\Big\}$$

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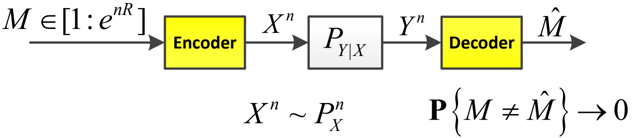


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- $C\left(P_X\right)$ also depends on $P_{Y|X}$.

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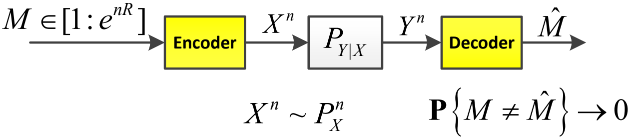


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- $C(P_X)$ also depends on $P_{Y|X}$.
- Without the constraint $P_{X^n} = P_X^n$, the Shannon capacity is

$$C(P_{Y|X}) = \max_{P_X} I(X;Y).$$

# Main Result

## Theorem

*We have*

$$C(P_X) = C_{\mathsf{GK}}(X;Y),$$

*where*

$$C_{\mathsf{GK}}(X;Y) := \sup_{f,g:f(X)=g(Y)} H(f(X)) = \sup_{V:V-X-Y-V} H(V)$$

*denotes the Gács-Körner (GK) common information between $X$ and $Y$ (under the distribution $P_X \times P_{Y|X}$).*

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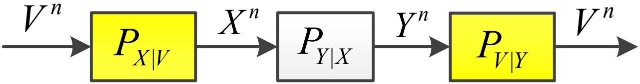
Note that

$$C_{\mathsf{GK}}(X;Y) \le I(X;Y), \quad \text{and} \quad \max_{P_X} C_{\mathsf{GK}}(X;Y) \le \max_{P_X} I(X;Y)$$

# Proof of Achievability: GK Mapping

$$C_{\mathsf{GK}}(X;Y) := \sup_{V : V - X - Y - V} H(V)$$

- The $n$ copy version of the single-shot Markov chain $V - X - Y - V$ is:

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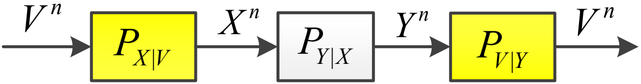


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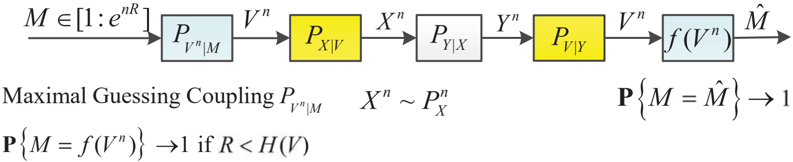
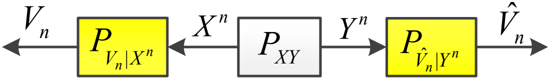
- Any GK common information $V^n$ can be transmitted losslessly
- However, $V^n$ is not a uniform random variable!




- Any rate $R < \sup_{V : V - X - Y - V} H(V) = C_{\mathsf{GK}}(X;Y)$ can be achieved

$$C_{\mathsf{GK}}(X;Y) := \sup_{V: V - X - Y - V} H(V)$$



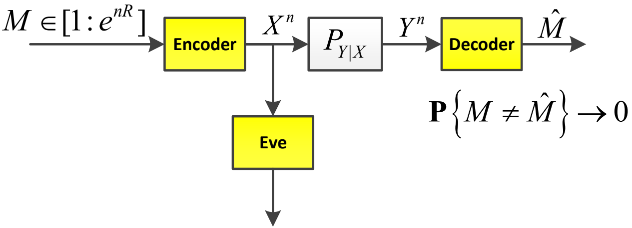

## Theorem (Gács-Körner (1973))

*For any* $\epsilon \in (0,1)$,

$$\lim_{n\to\infty} \sup_{(P_{V_n|X^n}, P_{\hat{V}_n|Y^n})} \left\{ \frac{1}{n} H(V_n) : \mathbb{P}\left(V_n \neq \hat{V}_n\right) \le \epsilon \right\} = C_{\mathsf{GK}}(X;Y)$$

- Our converse is a special case: $V_n$ is uniform

# Application to Perfect Stealth Communication




Being in transmission or not?

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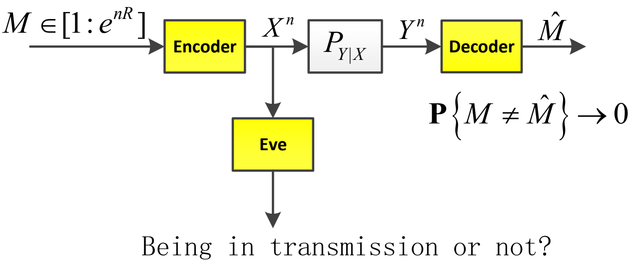


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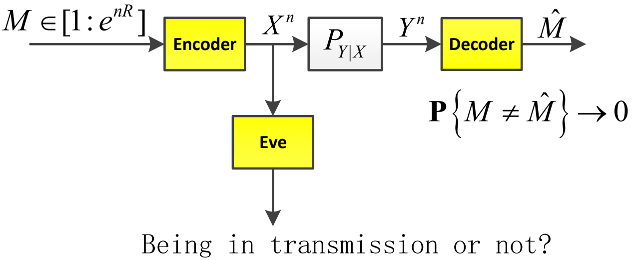


- When $M$ is not transmitted over the channel, Eve observes $X^n \sim P_X^n$, which can be regarded as pure noise
- To prevent Eve to detect the transmission, encoder and decoder should satisfy

$$X^n \sim P_X^n, \quad \text{and} \quad \lim_{n\to\infty} \mathbb{P}\left\{M_n \neq \widehat{M}_n\right\} = 0$$

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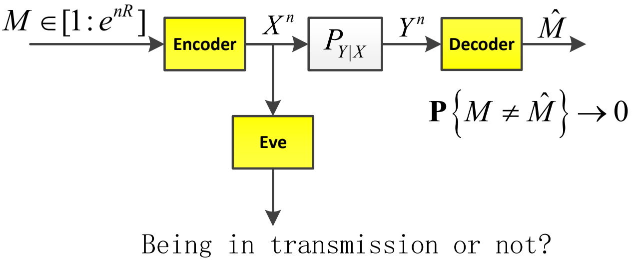


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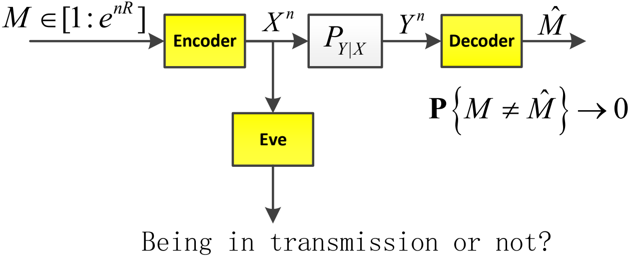
- Perfect stealth communication problem is equivalent to channel coding problem with input distribution constraint

# Application to Perfect Stealth Communication




**Theorem**

*Perfect stealth capacity is* $C_{\mathsf{GK}}(X;Y)$.

# Outline

# Summary

- In this work, we studied the maximal guessing coupling problem:
  - showed that it typically converge at least exponentially fast to 0 or 1.
  - applied this result to two new information-theoretic problems — channel capacity with input distribution constraint, and perfect stealth communication.

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- Open problem:
  - maximal guessing coupling problem for $H(X) = H(Y)$ but $P_X \neq P_Y$.
- Some other coupling problems can also be found the extended version of our paper

*Lei Yu and Vincent Y. F. Tan, "Asymptotic coupling and its applications in information theory," submitted to IEEE Trans. Inf. Theory, Dec. 2017. Available at arXiv:1712.06804.*

*Thank you for your attention!*