# Fast Beam Alignment via Pure Exploration in Multi-Armed Bandits

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#### June 29, 2022

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### The Beam Alignment Problem



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# The Beam Alignment Problem



Beam alignment (BA) is to ensure the transmitter and receiver beams are accurately aligned to establish a reliable communication link.

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# Some Fundamental Challenges in Beam Alignment



- To find optimal Rx/Tx pair, CSI corresponding to each pair is measured
- Frequency of each measurement is high
- High beam alignment latency

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# Some Fundamental Challenges in Beam Alignment



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Beam alignment latency will increase with the number of antennas at the receivers and transmitters.

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Main Contribution: Formulate and solve the beam alignment problem using ideas in pure exploration in the fixed-confidence setting.

Exploit structure in the beam alignment problem.

# System Model



Figure: A mmWave massive MISO system system.

• Massive mmWave MISO system: A base station (BS) equipped with *N* transmit antennas serves a single-antenna user

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Figure: A mmWave massive MISO system system.

- Massive mmWave MISO system: A base station (BS) equipped with *N* transmit antennas serves a single-antenna user
- Adopt the widely-used Saleh–Valenzuela channel model

$$\mathbf{h} = \beta^{(1)} \mathbf{a} \left( \theta^{(1)} \right) + \sum_{l=2}^{L} \beta^{(l)} \mathbf{a} \left( \theta^{(l)} \right)$$

$$\mathbf{a}_{\text{Amplitude}}$$
1 line-of-sight (LoS) path  $\geq$  L - 1 non-LoS (NLoS) paths

#### **Transmission Scheme**



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### **Transmission Scheme**



• Beam alignment phase: Fast beam alignment algorithm searches the optimal beam from a given codebook

$$\mathcal{C} \triangleq \{\mathbf{f}_k = \mathbf{a}(-1 + 2k/K) \mid k = 0, 1, \dots, K - 1\}$$

where the array response vector:

$$\mathbf{a}(x) = \frac{1}{\sqrt{N}} \left[ 1, e^{j\frac{2\pi}{\lambda}dx}, e^{j\frac{2\pi}{\lambda}2dx}, \cdots, e^{j\frac{2\pi}{\lambda}(N-1)dx} \right]^{\mathrm{H}} \in \mathbb{C}^{N \times 1}$$

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• Data transmission phase: BS transmits effective data using the selected **f**\*. Received signal at the user in time slot *t* is

$$y_t = \sqrt{p} \mathbf{h}^{\mathrm{H}} \mathbf{f}^* s_t + n_t,$$

# **Beam Alignment Phase**

• System Throughput Performance: Effective achievable rate

$$\textit{R}_{\text{eff}} \triangleq \left(1 - \frac{\textit{T}_{\text{B}}}{\textit{T}_{\text{D}}}\right) \log \left(1 + \frac{\textit{p} | \textbf{h}^{\text{H}} \textbf{f}^{*} |^{2}}{\sigma^{2}}\right)$$

Time to search for optimal beam  $T_{\rm B}$  to be minimized for high  $R_{\rm eff}$ 

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Measurement: The received signal power

### Relation to MABs and Properties



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**Property:** Let  $\mu = (\mu_1, \dots, \mu_K)$ , and let  $\mu_{(1)} \ge \mu_{(2)} \ge \mu_{(3)} \ge \dots \ge \mu_{(K)}$  be the sorted sequence of means.

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**Property:** Let  $\mu = (\mu_1, \dots, \mu_K)$ , and let  $\mu_{(1)} \ge \mu_{(2)} \ge \mu_{(3)} \ge \dots \ge \mu_{(K)}$  be the sorted sequence of means.

- 1. The means of the rewards associated with close-by arms are close.
- 2. The variance of the reward of an arm is linearly related to its mean

$$\sigma_k^2 = 2\mu_k \sigma^2.$$

#### Beam Codebooks Possess the Group Property

 $\frac{1}{d}$ -lower resolution beam codebook

• Constructed by grouping the nearby beams in the codebook  $\ensuremath{\mathcal{C}}$ 

$$\mathcal{C}_{(J)} \triangleq \left\{ \mathbf{b}_g = \sum_{k=J(g-1)+1}^{Jg} \mathbf{f}_k \ : \ g = 0, 1, \dots, G-1 \right\}$$

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Received power for beam b<sub>g</sub>

$$R_g = \rho |\mathbf{h}^{\mathrm{H}} \mathbf{b}_g|^2 + 2\sqrt{\rho} \Re(\mathbf{h}^{\mathrm{H}} \mathbf{b}_g n^*),$$

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Information of a set of beams can be obtained at each time step

# Problem Setup for Bandit Beam Alignment

#### Bandit BA Problem

- *K* base arms  $[K] \triangleq \{1, \dots, K\}$ : associated with the beam  $\mathbf{f}_k$
- {[*K*], *J*}: set of all non-empty consecutive tuples of length  $\leq J$ {[6], 2} = {{1}, {1, 2}, {2}, {2, 3}, {3}, {3, 4}, {4}, {4, 5}, {5, 6}, {6}}
- (*K*, *J*)-super arm: each tuple in {[*K*], *J*}, associated with a "grouped beam" b<sub>g</sub> ∈ C<sub>(J)</sub>.

In time step t

• Choose an action (or a (K, J)-super arm)  $A(t) \in \{[K], J\}$ 

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- Observe the reward

$$R(t) = F\left(\sum_{k \in \mathcal{A}(t)} \mathbf{f}_k, \boldsymbol{\rho}, \mathbf{h}, n_t\right)$$

where  $F(\mathbf{f}, p, \mathbf{h}, n) = p |\mathbf{h}^{H} \mathbf{f}|^{2} + 2\sqrt{p} \Re(\mathbf{h}^{H} \mathbf{f} n^{*})$ 

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• Each observed reward *R*(*t*) also follows a heteroscedastic Gaussian distribution.

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Algorithm: 
$$\pi := \{(\pi_t)_t, \tau^{\pi}, \psi^{\pi}, J\}$$

- Sampling rule π<sub>t</sub>: Determines the ([K], J)-super arm A(t) to pull at time step t based on the observation history and the arm history {A(1), R(1), A(2), R(2), ··· , A(t 1), R(t 1)}
- Stopping rule: Leads to a stopping time  $\tau^{\pi}$  satisfying  $\mathbb{P}(\tau^{\pi} < \infty) = 1$
- Recommendation rule  $\psi^{\pi}$ : Outputs a base arm  $k^{\pi} \in [K]$ .

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Aim: Use as few samples as possible to output a guess of the optimal arm with probability at least  $1 - \delta$ .

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# Some Notations for the General Lower Bound

• Heteroscedastic Gaussian bandit instance:

$$\nu = \left\{ \mathcal{N}(\mu_1^\nu, 2\mu_1^\nu \sigma^2), \dots, \mathcal{N}(\mu_K^\nu, 2\mu_K^\nu \sigma^2) \right\}$$

• Optimal arm  $A^*(\nu)$ :

$$\mu^{\nu}_{\mathbf{A}^*(\nu)} = \operatorname*{argmax}_{k \in [K]} \mu^{\nu}_k$$

Set of probability distributions

$$\mathcal{W}_{K} := \left\{ \mathbf{w} \in \mathbb{R}_{+}^{K} : \sum_{k=1}^{K} w_{k} = 1 \right\}$$

Alternative Set

$$\mathsf{Alt}(\nu) \coloneqq \{ \mathsf{u} \in \mathcal{V} : \mathcal{A}^*(\mathsf{u}) \neq \mathcal{A}^*(\nu) \}$$

# **General Lower Bound**

#### Theorem

For any  $(\delta, J)$ –PAC algorithm where  $\delta \in (0, 1)$ , it holds that

$$\mathbb{E}_{\pi}[\tau] \geq \boldsymbol{c}^{*}(\nu) \ln \left(\frac{1}{4\delta}\right),$$

where

$$c^*(\nu)^{-1} = \sup_{\mathbf{w}\in\mathcal{W}_K} \inf_{\mathbf{u}\in\mathsf{Alt}(\nu)} \sum_{k=1}^K w_k D_{\mathrm{HG}}(\mu_k^{\nu}, \mu_k^{\mathrm{u}})$$

and the heteroscedastic relative entropy is

$$D_{\rm HG}(\mu_i, \mu_j) = \frac{1}{2} \ln \left(\frac{\mu_j}{\mu_i}\right) + \frac{\mu_i}{2\mu_j} + \frac{(\mu_j - \mu_i)^2}{4\mu_j \sigma^2} - \frac{1}{2}$$

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# Two-Phase Heteroscedastic Track & Stop (2PHT&S)

#### Main Idea

to exploit the **prior knowledge** which have not been considered by existing bandit-based beam alignment algorithms:

- Close-by correlation
- Heteroscedasticity
- Group property

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**Phase I**: Search for the optimal super arm w.p.  $\geq 1 - \delta_1$ 

Image: A matrix and a matrix



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- At each time,
  - Choose one super arm (beam group) by the sampling rule in a new Heteroscedastic Track and Stop (HT&S) algorithm



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  - Use the grouped beam to transmit the pilot symbols and observe the grouped reward *R*<sub>g</sub>(*t*).



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  - Choose one super arm (beam group) by the sampling rule in a new Heteroscedastic Track and Stop (HT&S) algorithm
  - Use the grouped beam to transmit the pilot symbols and observe the grouped reward *R*<sub>g</sub>(*t*).
- Select the optimal super arm

$$g^* = \underset{g \in [G]}{\operatorname{argmax}} \mathbb{E}[R_g(t)].$$





**Phase II**: Search for the optimal base arm w.p.  $\geq 1 - \delta_2$ 

 Construct a base arm set (including the optimal super arm and its neighboring super arm)



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- Construct a base arm set (including the optimal super arm and its neighboring super arm)
- Search for the optimal base arm in the base arm set using the HT&S algorithm

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# HT&S: An Improved Track & Stop Algorithm

• Sampling Rule: Estimate the number of times each arm should be sampled

$$Q(t) = \begin{cases} \operatorname{argmin}_{i} T_{i}(t-1), & \text{if } \min_{i} T_{i}(t-1) \leq \sqrt{t}, \\ \operatorname{argmax}_{i} t \hat{w}_{i}^{*}(t-1) - T_{i}(t-1), & \text{otherwise.} \end{cases}$$

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• Stopping Rule: Stop when the number of pulls of all arms meet a certain requirement

Threshold to be  $\beta(t, \delta, \alpha) = \ln(\alpha t/\delta)$ , and the stopping rule is

 $\tau_{\delta} = \min \{ t \in \mathbb{N} : \mathbf{Z}(t) \ge \beta(t, \delta, \alpha) \}.$ 

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 Heteroscedasticity: Considered explicitly in the calculations of estimated reward ŵ<sup>\*</sup><sub>i</sub>(t - 1) and statistic Z(t)

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# Time Complexity Analysis of 2PHT&S

#### Theorem

Let the means of the super and base arms be respectively

$$\begin{split} \mathbf{s} &\coloneqq \{\mathcal{N}(\mu_1^{\mathrm{s}}, 2\mu_1^{\mathrm{s}}\sigma^2), \dots, \mathcal{N}(\mu_G^{\mathrm{s}}, 2\mu_G^{\mathrm{s}}\sigma^2)\} \quad \text{and} \\ \mathbf{b} &\coloneqq \{\mathcal{N}(\mu_{\mathcal{S}_f(1)}^{\mathrm{b}}, 2\mu_{\mathcal{S}_f(1)}^{\mathrm{b}}\sigma^2), \dots, \mathcal{N}(\mu_{\mathcal{S}_f(2J)}^{\mathrm{b}}, 2\mu_{\mathcal{S}_f(2J)}^{\mathrm{b}}\sigma^2)\} \end{split}$$

where

$$\boldsymbol{\mu}_{g}^{\mathrm{s}} = \boldsymbol{\rho} \left| \mathbf{h}^{\mathrm{H}} \Big( \sum_{k \in \mathcal{S}_{g}} \mathbf{f}_{k} \Big) \right|^{2}.$$

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where

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$$\boldsymbol{\mu}_{g}^{\mathrm{s}} = \boldsymbol{\rho} \left| \mathbf{h}^{\mathrm{H}} \Big( \sum_{k \in \mathcal{S}_{g}} \mathbf{f}_{k} \Big) \right|^{2}.$$

Under 2PHT&S, we have

$$\limsup_{\delta \to 0} \frac{\mathbb{E}[\tau^{2 \textit{PHT&S}}]}{\ln(1/\delta)} \leq \textit{C}_{s} + \textit{C}_{b},$$

where  $C_{\rm s}$  and  $C_{\rm b}$  represent time complexities of Phases I and II resp.

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# **Simulation Results**

#### **Experiment Setup**

- Massive mmWave MISO system
- Base station equipped with 64 transmit antennas serving a single-antenna user
- Size of codebook is set as K = 128.

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#### **Baseline Algorithms**

- Original Track-and-Stop (T&S) algorithm (Garivier et al. 2016)
- Two-Phase Track-and-Stop (2PT&S) algorithm
- Heteroscedastic Track-and-Stop (HT&S) algorithm

A. Garivier and E. Kaufmann, "Optimal best arm identification with fixed confidence." in PMLR, 2016, pp. 998-1027.

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June 29, 2022

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### Numerical Simulations on Synthetic Data



Figure: Mean of the reward of each base arm and super arm (p = 10 dBm).

Table: Average time complexities over 100 experiments when  $\delta = 0.1$ 

Power	4	6	8	10	12
T&S	1154.3 ±338.7	654.6 ±212.1	382.5 ±129.6	209.4±68.6	133.7 ±8.9
HT&S	473.2 ±275.5	$271.4 \pm 143.4$	175.6 ±69.2	133.2 ±24.1	123.9 ±6.5
2PT&S	$206.2 \pm \scriptstyle 60.4$	120.2 ±35.0	68.4 ±19.4	49.1 ±4.6	45.2 ±1.1
2HPT&S	84.3 ±41.5	$58.0 \pm 19.6$	48.4 ±6.3	45.5 ±1.6	45 ±0

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2PT&S	$206.2 \pm \scriptstyle 60.4$	$120.2 \pm 35.0$	68.4 ±19.4	49.1 ±4.6	45.2 ±1.1
2HPT&S	84.3 ±41.5	$58.0 \pm 19.6$	48.4 ±6.3	45.5 ±1.6	45 ±0

Experiments on real data for a practical scenario in a city available in the longer version of our paper.

• Formulate the beam alignment problem as a multi-armed bandit problem under the pure exploration setting

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- Exploited the structure and properties of the beam alignment problem to derive an algorithm 2PHT&S with a reduced time complexity
- Simulations demonstrate superior performances over benchmarks